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ABSTRACT

UniSuper[1] has developed a risk budgeting system that measures deviations from the Fund's strategic asset allocation. The paper presents the mathematics used, a case study, and broad conclusions potentially applicable to institutional investors.

Keywords: risk budgeting; factor analysis; marginal and proportional contribution to risk; return attribution; ex-ante alpha; collinearity; reverse optimisation.

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1. INTRODUCTION

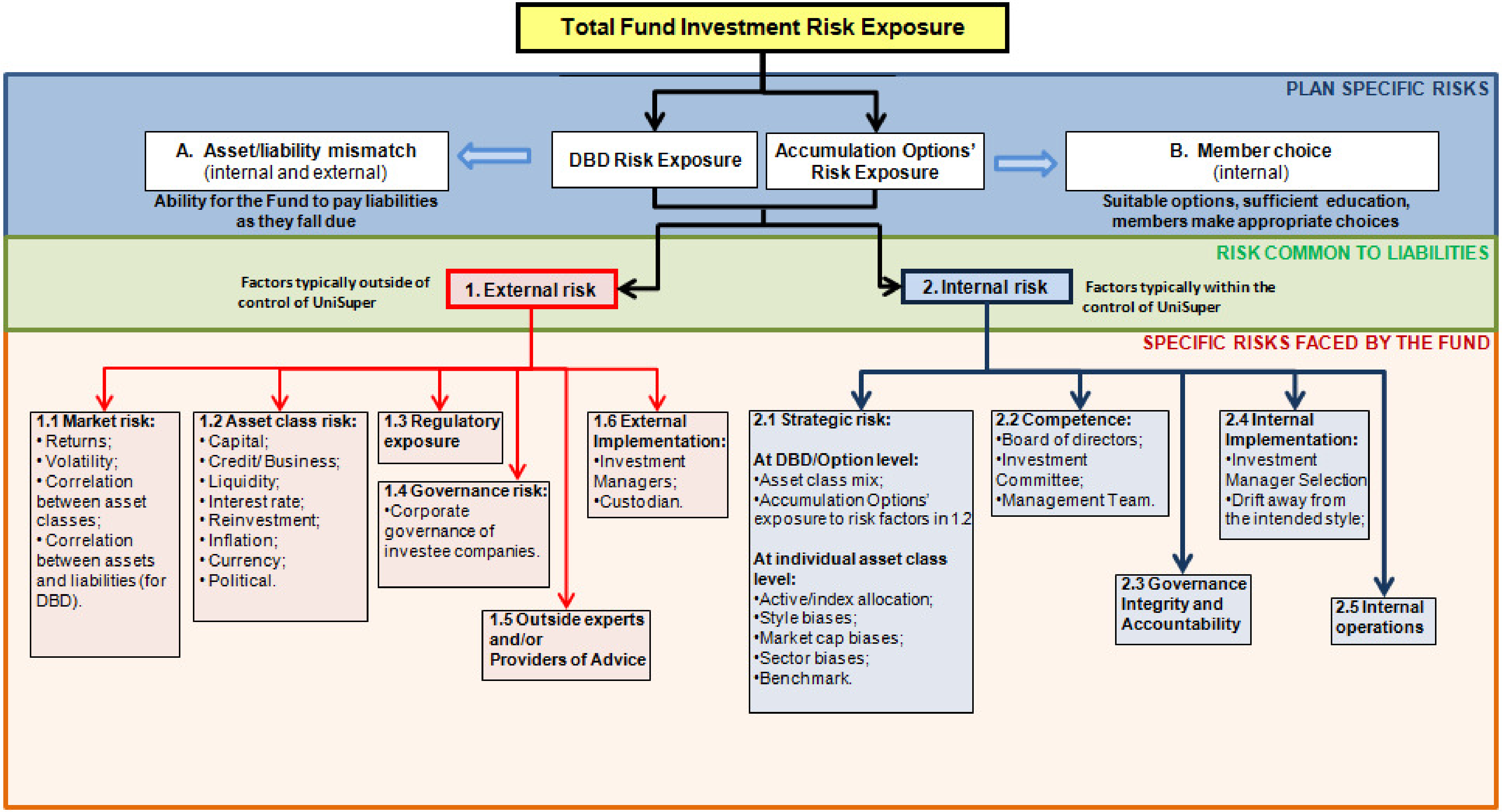
At its simplest, risk budgeting is the process of setting a target level of risk to be accepted at the portfolio level, and allocating this risk across a number of investments in the most efficient manner in order to maximise returns whilst containing risk within agreed targets. At the sector or asset class level, risk budgets are typically set in terms of a tracking error target for the overall sector (i.e. how much volatility around an appropriate sector benchmark would the Fund be prepared to accept in order to try and increase returns). It is important to note that risk budgeting primarily provides investors with a framework for discussion and analysis. It does not have to be (and indeed in the authors' opinion, should not be) applied in a prescriptive fashion.

This paper focuses on a single component of overall fund level risk management. UniSuper has derived a formal Investment Risk Management Policy (IRMP) that sets out the Fund's risk management philosophy, identifies key internal and external risks faced by the Fund, and the manner with which these risks are managed (UniSuper, 2005). The Fund's risk management initiatives span the public and private market areas, as well as the Fund's strategic tilting and governance arrangements. The risk classification framework and risk management initiatives underpinning UniSuper's IRMP are graphically represented in the chart overleaf.

UniSuper's risk budgeting approach has been developed over a number of years, and focuses on measuring and managing the active risk accepted by the Fund (i.e. item 2.1 in the chart overleaf). Details relating to UniSuper's management of other components of risk within the Fund are outside the scope of this paper. Much of the work undertaken by UniSuper is built up from prior work on risk budgeting (for example, Mina (2007) provides an outline of a risk budgeting framework, which formed the basis of much of the work in this paper; Litterman (2003b) describes a practical methodology for institutional investors; while Scherer (2000), Kozun (2001), Sharpe (2002), Banz (2003), de Bever (2003) and Berkelaar (2006) provide an overview of risk budgeting for institutional investors). However, the authors extended prior findings by:

- removing the simplifying assumption that excess returns between managers are uncorrelated (it is common for managers employing similar investment styles to perform in a correlated manner);
- introducing the idea that to justify active risk, one needs to exceed a hurdle return in excess of 0% (described in more detail in the Appendix section A1.7); and
- developed a method to help overcome the difficulties inherent with multiple collinearity (described in more detail in the Appendix section 4.2 and A1.2.4).

CHART 1. Schematic representation of the risks faced by UniSuper



Source: UniSuper, 2005

The above chart summarises the risks faced by UniSuper. The Fund's risk budgeting approach measures active risk accepted by the Fund, as highlighted in box 2.1.

Due to the computational complexity of the Fund's risk budgeting process, UniSuper has developed an in-house risk budgeting and factor analysis program, branded 'The UniSuper Risk Budgeting and Optimisation System' (TURBOs). This paper outlines the objectives and logic underlying TURBOs, provides practical commentary as well as a worked example for the Fund as at 30 June 2008.

2. MATHEMATICAL OVERVIEW

TURBOs monitors the extent to which the Fund deviates from its Strategic Asset Allocation (SAA). UniSuper can invest passively and broadly match the beta exposures expected from each asset class. The extent to which the Fund employs active management, and the amount by which the Fund deviates from the passive benchmark position, represents a source of risk to the Fund. The objective behind TURBOs is to identify and quantify this source of risk.

UniSuper predominantly invests via fund managers, but the Fund does hold some investments directly. As such, for the purpose of this paper, we use the term manager and investment interchangeably.

TURBOs has been designed so as to:

- Determine how each investment or manager generates their returns;
- Identify the market factor exposures for each manager and aggregate these to determine overall Option factor exposures;
- Assess the expected future alpha (or excess return above benchmark) for each manager; and
- Ensure that the Fund's active risk is allocated appropriately between managers (i.e. such that most of the Fund's active risk lies with managers who are expected to outperform their benchmarks).

In order to meet the objectives, six processes need to be computed. In particular, TURBOs:

1. Attributes each manager's returns between a series of market factor exposures (i.e. a beta component) and an observed ex-post (or historic) alpha component. This step is resolved using factor analysis and multiple regression;
2. Determines the ex-post total risk (or volatility) and tracking error for each Option, and assess the marginal and proportional contribution to that risk, from each manager;
3. Uses Bayesian techniques to determine an ex-ante (or forecast) estimate of each manager's alpha;
4. Assesses the extent to which each Option's beta exposure differs to the Fund's SAA Benchmarks;
5. Sets a minimum active risk target for each Option and assess the extent to which the hurdle is expected to be achieved; and
6. Employs reverse optimisation to confirm whether the weight assigned to each of the Fund's managers is consistent with the expected performance of the manager.

Each of these six processes are described mathematically below. *The equations presented are developed in Appendix 1.* To assist the reader, we provide a glossary of notation in Appendix 2. Note that equation references provided in parenthesis after each equation, relate to the order in which the equation is developed in Appendix 1.

2.1 Factor Analysis

Regression techniques are utilised to derive *ex-post* estimates of the source of each manager's total return. Each manager's return consists of a beta component along with an alpha (or return in excess of market factors) component. Specifically: $\mu_{i,t}^{a_i} = \hat{\alpha}_{i,t}^{a_i} + \sum_{k=1}^{K} \hat{\beta}_{i,k,t} F_{k,t} + \varepsilon_{i,t}$ ...(2.1.5)

Where

$\mu_{i,t}^{a_i}$	Denotes the observed return (before tax, net of fees) from manager *i* at time *t*, where manager *i* invests in asset class a_i.
$\hat{\alpha}_{i,t}^{a_i}$	Denotes the estimated ex-post return in excess of market factors, for manager *i* at time *t,* where manager *i* invests in asset class a_i.
a_i	Denotes manager *i*'s asset class.
K	Represents the number of all applicable factors. Typical factors are returns on stock indices, interest rates, volatility, the risk free rate of return etc.
$\hat{\beta}_{i,k,t}$	Denotes the estimated beta factor describing the sensitivity between manager *i*'s exposure to factor *k* at time *t*.
$F_{k,t}$	Denotes the observed returns from factor *k* at time *t*.
$\varepsilon_{i,t}$	Denotes the manager's residual error term or unexplained returns, with a zero mean.

The alpha and beta parameters from equation 2.1.5, are estimated using the techniques outlined in section 2 of Appendix 1 (refer equation 2.3.2). Once each manager's alpha estimate and beta factors are obtained, standard statistical techniques are used to test the goodness of fit.

2.2 Marginal and Proportional Contribution to Risk

Each Option's risk (or volatility) can be estimated by considering historic data. The total ex-post variance of returns for each Option is defined as $\hat{\sigma}_{o,t}^2$ and is derived in equation 3.2: $\hat{\sigma}_{o,t}^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{i,t}^{o,a_i} w_{j,t}^{o,a_j} \hat{\sigma}_{ij,t}$*...(3.2)*

Where

$w_{i,t}^{o,a_i}$	Denotes the manager *i*'s weight in Option *o* at time *t* (i.e. percentage holding, weighted by Funds Under Management).
$\hat{\sigma}_{ij,t}$	Denotes the estimated covariance of returns (gross of tax, net of fees) between manager *i* and manager *j* at time *t,* with an adjustment to allow for manager *i* and *j*'s auto-correlation.
N	Denotes the total number of managers spanning all asset classes.

The covariance between each Manager's returns explicitly allows for auto-correlation, and the formula for the covariance is presented in equation 3.1. Each manager's marginal contribution to risk amounts to: $\frac{\delta \hat{\sigma}_{o,t}^2}{\delta w_{i,t}^{o,a}} = 2\sum_{j=1}^{N} w_{j,t}^{o,a_j} \hat{\sigma}_{ij,t}$,which is derived in equation 3.4.

2.3 Ex-Ante Alpha Estimation

Each manager's ex-post alpha is estimated during the factor analysis calculation. However, ex-post data is not a reliable guide to estimate future alpha expected to be generated by managers. To account for UniSuper's internal view as to each manager's skill, a Bayesian approach is required. The authors adapted the Black-Litterman Model (BLM) so as to derive ex-ante alpha estimates for each manager. The workings behind the adaptation of the BLM are discussed in Section 4 of Appendix 1, and derived in equation 4.2.9:

$$\hat{\hat{\alpha}} = \left[\left(\tau\hat{\hat{\Psi}}\right)^{-1} + \overline{\Omega}_A^{\ -1} \right]^{-1} \left[\left(\tau\hat{\hat{\Psi}}\right)^{-1} \overline{\mathrm{A}} + \ \overline{\Omega}_A^{\ -1} \overline{Q}_A \right] \text{...(4.2.9)}$$

Where:

$\hat{\hat{\alpha}}$	Denotes the vector of each manager's expected ex-ante alpha returns. Each element of the vector is given the symbol $E[\hat{\alpha}_{i,t}^{a_i}]$.
$E[\hat{\alpha}_{i,t}^{a_i}]$	Denotes the ex-ante expected alpha from manager *i*, who operates in asset class a_i at time *t*.
$\overline{\mathrm{A}}$	Denotes the vector of observed ex-post alpha or excess returns for each manager, derived using factor analysis from equation 2.1.5. Note that this vector can alternatively represent the equilibrium excess return for each manager (which would then be a null vector, and the term $\left[\left(\tau\hat{\hat{\Psi}}\right)^{-1}\mathrm{A}\right]$ would be removed from equation 4.2.9.)
$\hat{\hat{\Psi}}$	Denotes the covariance matrix of each manager's ex-post returns in excess of their factor exposures.
τ	Represents the scaling factor applied to $\hat{\hat{\Psi}}$ which measures the uncertainty of the historic data.
$\overline{Q}_A$	Represents a vector containing the investor's view of the expected alpha generated from each manager along the diagonal elements.
$\overline{\Omega}_A$	Represents a square matrix containing the investor's confidence in its view of the expected alpha generated from each manager along the diagonal elements.

2.4 Return Attribution

By combining and weighting the Fund's exposure to all *N* managers we can derive the ex-ante expected return for Option (*o*) and attribute this return between a variety of sources.

Within the set of available beta factors $\{F_k\}_{k=1}^{K}$ there is a sub-set of *M* factors that relate to the Fund's benchmark for each asset class (as an example, for the Australian Equity asset class, UniSuper's current benchmark is the ASX 300 Accumulation index – which in turn is a potential beta factor for all Australian equity managers).

Let $\{BM^*{}_m\}_{m=1}^{M}$ denote the set of strategic benchmark factors, for each of the Fund's *M* asset classes. Then one can determine the manner in which each beta factor maps onto the Fund's strategic benchmark factors as follows:

$$F_{k,t} = RP_k + \sum_{m=1}^{M} \hat{\gamma}_{m,k}\ BM^*_{m,t} + \nu_{k,t} \ldots(5.1)$$

Where

RP_k	Denotes the risk premium available from Factor *k* which cannot be attributed to one of the Fund's benchmarks.
$\hat{\gamma}_{m,k}$	Denotes the estimated sensitivity between factor *k* and the m^{th} asset class benchmark.
$\nu_{k,t}$	Denotes the residuals of the regression solution, with a zero mean.

The risk premia and the $\hat{\gamma}_{m,k}$ coefficients are estimated in the same manner that was used to derive the factors from equation 2.1.5.

By combining equations 2.1.5, 4.2.9 and 5.1, we can derive the expected return for each option expressed as a function of:

a) The weighted average of each manager's expected ex-ante alpha;

b) The weighted average exposure to the Fund's SAA benchmarks;

c) Extraneous beta risk premia from factors that differ to the Fund's SAA benchmark; and

d) An error term.

If R_t^o denotes the ex-post (or historic) return from Option o at time t, then:

$$R_t^o = \left\{ \sum_{i=1}^{N} w_{i,t}^{o,a_i} \hat{\alpha}_{i,t}^{a_i} \right\} + \left\{ \left[\sum_{i=1}^{N} \sum_{k=1}^{K} \sum_{m=1}^{M} w_{i,t}^{o,a_i} \hat{\gamma}_{m,k} \hat{\beta}_{i,k,t} \, BM_{m,t}^{*} \right] + \left[\sum_{i=1}^{N} \sum_{k=1}^{K} w_{i,t}^{o,a_i} \hat{\beta}_{i,k,t} RP_k \right] \right\} + \varepsilon$$

Where $\varepsilon = \left\{ \sum_{i=1}^{N} \sum_{k=1}^{K} w_{i,t}^{o,a_i} \hat{\beta}_{i,k,t} v_{k,t} + \sum_{i=1}^{N} w_{i,t}^{o,a_i} \varepsilon_{i,t} \right\}$ ....(5.5)

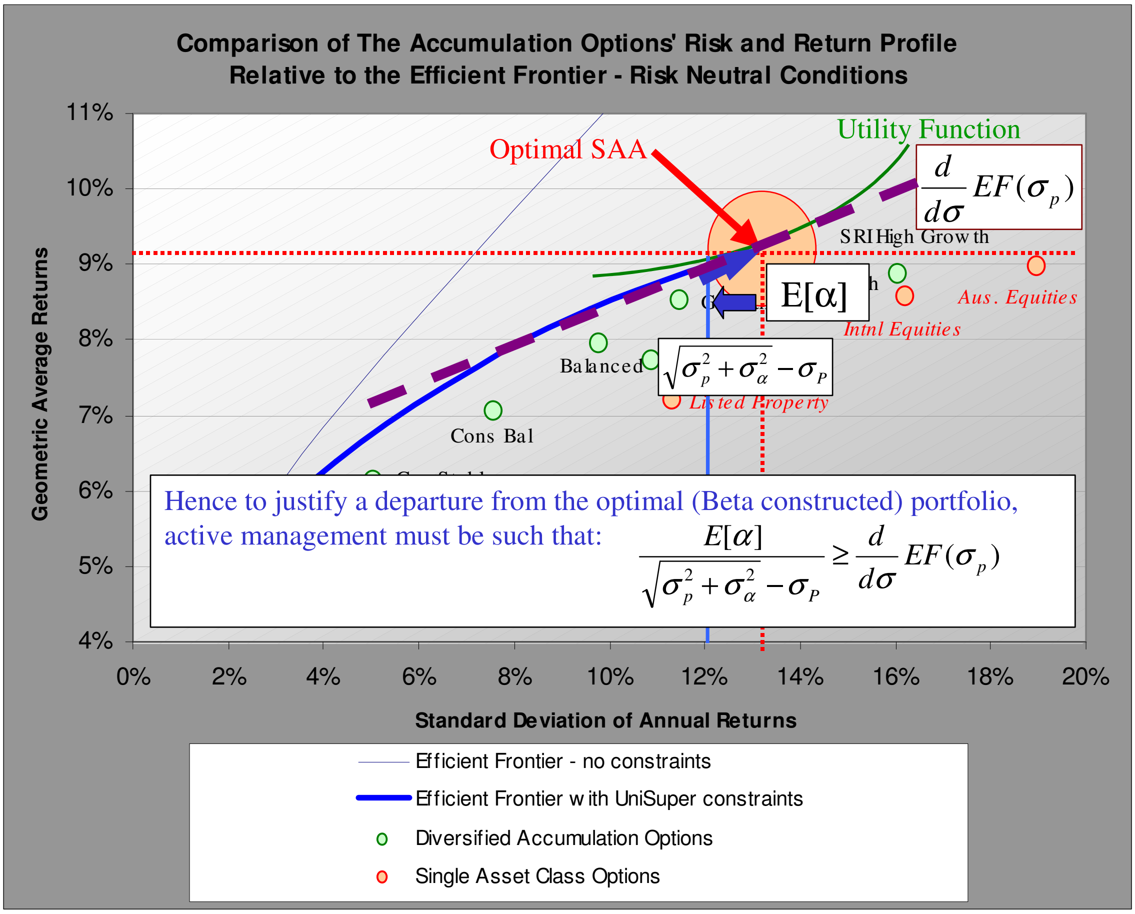
The above equation is central to the Fund's factor analysis. In particular:

- The term $\left\{ \sum_{i=1}^{N} w_{i,t}^{o,a_i} \hat{\alpha}_{i,t}^{a_i} \right\}$ denotes the weighted average ex-post alpha observed from the Fund's managers.
- The term $\left[\sum_{i=1}^{N} \sum_{k=1}^{K} \sum_{m=1}^{M} w_{i,t}^{o,a_i} \hat{\gamma}_{m,k} \hat{\beta}_{i,k,t} BM_{m,t}^{*} \right]$ denotes the weighted average exposure to the Option's SAA benchmark, which will be contrasted to the Option's actual SAA;
- While $\left[\sum_{i=1}^{N} \sum_{k=1}^{K} w_{i,t}^{o,a_i} \hat{\beta}_{i,k,t} RP_k \right]$ denotes the Option's exposure to other known beta sources (such as credit risk, value, small cap biases, sector bets etc); and
- The final term $\left\{ \sum_{i=1}^{N} \sum_{k=1}^{K} w_{i,t}^{o,a_i} \hat{\beta}_{i,k,t} v_{k,t} + \sum_{i=1}^{N} w_{i,t}^{o,a_i} \varepsilon_{i,t} \right\}$, represents the error or residuals of the various regression estimates (with a zero mean). The distribution is critically examined to ensure that it is sufficiently normally distributed. Evidence of extreme kurtosis or skewness, could be cause for concern.

2.5 Risk Budgeting

Risk budgets have traditionally been derived by most practitioners with reference to a maximum permissible tracking error. This approach is appropriate for asset managers whose mandates are often specified in terms of tracking error limitations. However, the concern with this approach for institutions with guaranteed liabilities is that there is no direct interaction between the maximum tracking error and the Fund's liabilities. In addition, the choice of an appropriate tracking error budget is subjective. As a result, the authors have derived an alternative approach to risk budgeting.

The Fund sets its SAA so as to best meet the investment objectives (for the Accumulation Options) and pay liabilities as they fall due (for the Defined Benefit Division). Hence any deviation from the Fund's SAA represents a source of risk to the Fund. Specifically, introducing active management adds risk to the Fund. The marginal increase to risk is only justifiable if the Fund's expected (or *ex-ante*) alpha exceeds the benefit that could be obtained by changing the Fund's SAA benchmarks, and moving along the Fund's constrained efficient frontier. This idea (discussed in more detail in Section 7 of Appendix 1 and graphically in the chart below) provides an inequality that is used in our risk budgeting formulation, namely that each option's *ex-ante* alpha needs to exceed a minimum hurdle to justify a departure from beta allocations.



In section 7 of the Appendix, we derive an inequality linking the ex-ante alpha to the Fund's SAA, and its constrained efficient frontier:

$$\frac{\bar{w}'\hat{\bar{\alpha}}}{\sqrt{\sigma_o^2+\bar{w}'\hat{\bar{\Psi}}\bar{w}}-\sigma_o} \ge \frac{\delta EF(\sigma_O)}{\delta\sigma} \quad ...(7.1)$$

Where σ_o^2 represents Option o's variance, based on the SAA long term weights invested in the benchmark for each asset class and $\frac{\delta EF(\sigma_O)}{\delta\sigma} = \left.\frac{\delta EF}{\delta\sigma}\right|_{\sigma=\sigma_O}$ equals the derivative or slope of the constrained efficient frontier, with respect to the volatility of the frontier, solved when $\sigma=\sigma_O$. Note also that $\hat{\bar{\Psi}}$ in this equation denotes the covariance matrix of each manager's returns in excess of the benchmark.

The table below summarises the minimum hurdle for each option, along with the minimum excess required alpha to justify using active management as opposed to increasing risk by changing the Fund's SAA.

Table 1: Minimum required alpha for differing Accumulation Options

Accumulation Option (o)	Derivative of constrained efficient frontier $\frac{\delta EF(\sigma_O)}{\delta\sigma}$	Minimum required alpha ($\overline{w}'\hat{\alpha}$) under differing Option tracking error levels		
		1.00%	**1.50%**	**2.00%**
Cash	**3.81**	3.81%	5.72%	7.62%
Capital Stable	**0.45**	0.45%	0.68%	0.90%
Conservative Balanced	**0.34**	0.34%	0.51%	0.68%
Balanced	**0.28**	0.28%	0.42%	0.56%
Growth	**0.26**	0.26%	0.39%	0.52%
High Growth	**0.14**	0.14%	0.21%	0.28%

As can be seen in the above table, each Option's required minimum *ex-ante* alpha varies according to the curvature of the efficient frontier as well as the tracking error for the Option, relative to the Option's benchmark. Hence if the observed option risk is 2% (say) higher than that which would have occurred had UniSuper invested passively for the Balanced Option, then the minimum required excess return for the Balanced Option (to justify a departure from the Option's SAA) would amount to 0.56%.

TURBOs computes the *ex-ante* alphas, using equation 4.2.9, together with the *ex-post* tracking error for each Option and contrasts these to the minimum hurdle (provided by equation 7.1) to ensure that the Fund remains comfortable with its manager line-up.

2.6 Reverse Optimisation

Equation 7.1 can be adapted to obtain an optimal manager line-up (denoted by $\overline{w}^*$). Such a portfolio would have the highest risk-adjusted return whilst simultaneously meeting the minimum hurdles derived in section 3.5 above. The adaptation is derived in section 7 of the Appendix.

$$\overline{w}^* = \frac{2\sigma_o \frac{\delta EF(\sigma_O)}{\delta\sigma}\hat{\Psi}^{-1}\hat{\alpha}}{\left(\frac{\delta EF(\sigma_O)}{\delta\sigma}\right)^2 - \hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}} \quad \ldots(7.4)$$

Once $\overline{w}^*$ has been calculated, UniSuper contrasts the Fund's actual manager weights per Option to that derived by TURBOs, to determine whether the Fund remains comfortable with the current manager line up. Note though that $\overline{w}^*$ is calculated separately for each asset class and not for the overall fund, and that $\hat{\Psi}$ denotes the covariance matrix of each manager's returns in excess of their benchmark.

3. CASE STUDY

In this section we present the findings from an analysis undertaken using UniSuper's manager line up as at 30 June 2008, along with the Fund's historic data. The case study provides detail on the Fund's Australian shares managers, along with summarised findings for other asset classes. The analysis considers monthly historic returns over the three years ending 30 June 2008.

Considerations relating to practical adjustments and allowances required for alternative assets as well as for details pertaining to the calibration of the BLM are discussed in section 4 of this paper.

3.1. Factor Return Attribution

The first process utilised by TURBOs is to attribute each managers' returns between market factor exposures (i.e. the beta component) and an observed *ex-post* (or historic) alpha component. The estimated factor exposures and alpha are determined using multiple regression. For each manager's alpha estimate and beta factors selected, statistical measures are provided to test the goodness of fit. TURBOs assesses each manager in turn, utilising a backward elimination algorithm, to find an optimal fit to the market factors. The algorithm eventually selected by the authors is described in Appendix 1. The factors used include index returns by market capitalisation, style (value and growth), momentum and sectors, but remove the impact of higher order factor correlations, as described in Appendix 1. The table overleaf displays each of UniSuper's Australian shares manager's exposures to the various beta factors. A positive allocation indicates a bias to the factor, whilst a negative allocation suggests a bias away from the factor. The last column provides the aggregated exposures for UniSuper's Australian shares portfolio as at 30 June 2008.

The primary beta used is the ASX 300 (the benchmark for the portfolio). UniSuper's overall Australian Shares portfolio had a beta of 0.97. The portfolio is currently structured with a large beta exposure overlaid by meaningful alpha. The significant beta exposure is consistent with the largely long-only approach adopted by the Fund and the requirement on incumbent managers to remain as close to fully invested as possible. Pleasingly, there is minimal residual kurtosis, indicating that over the period analysed, there have been few extreme moves relative to benchmark. At an overall portfolio level, TURBOs indicates that the portfolio has a moderate bias to small caps. This positioning is consistent with UniSuper's expectations. From a style perspective, the analysis displays negative exposures to both value and growth (thus broadly style neutral). However, the authors believe that the exposure to value is being partly understated and rather being indirectly reflected through an implied sectoral bias to financials. Nothwithstanding this, the analysis confirms that the portfolio does not have any unintended style tilts. Overall, the analysis suggests that the portfolio is constructed without significant aggregate factor exposures and is well diversified, in that the portfolio has captured the bulk of the ASX 300 beta. Pleasingly, there is a solid level of underlying ex-post alpha (i.e. alpha remaining after all beta factors are removed) of 1.6% p.a..

Asset Sub Class	**Style 1**		**Style 2**	**Style 3**							**Style 4**		**Style 5**				**Style 6**	**Total**
Manager Code	**Manager A**	**Manager B**	**Manager C**	**Manager D**	**Manager E**	**Manager F**	**Manager G**	**Manager H**	**Manager I**	**Manager J**	**Manager K**	**Manager L**	**Manager M**	**Manager N**	**Manager O**	**Manager P**	**Manager Q**	
Weight	**7.6%**	**6.6%**	**17.3%**	**6.1%**	**9.5%**	**5.3%**	**7.2%**	**7.0%**	**2.9%**	**6.3%**	**1.7%**	**1.3%**	**0.7%**	**6.1%**	**6.8%**	**7.5%**	**5.3%**	**100%**
Manager ex-Post Alpha p.a.	4.4%	-2.5%	-0.2%	1.6%	-0.6%	1.6%	0.5%	1.0%	-2.8%	0.5%	11.4%	5.6%	-4.2%	3.9%	7.3%	4.6%	1.6%	1.6%
ASX300 Accum. Index	**94.3%**	**111.5%**	**99.8%**	**101.9%**	**103.2%**	**112.7%**	**94.0%**	**83.1%**	**108.9%**	**96.6%**	**85.6%**	**110.1%**	**124.6%**	**86.1%**	**82.7%**	**84.6%**	**112.7%**	**96.7%**
12-Month Momentum	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-4.8%	-	-	-0.3%
3-Month Momentum	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-9.6%	-	-	-0.6%
6-Month Momentum	-	-	-	-	-	-	-	-	-	-	-	-	-	-8.4%	-	-	-	-0.5%
Consumer Discretionary	9.8%	-	-	8.9%	-	-	-	-	20.5%	-	-	-	-	-	-	17.5%	-	3.2%
Consumer Staples	-	-	-	-	-	-	11.6%	-	-	-	-	-	-37.1%	-	-	-	-	0.6%
Energy	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Financials	46.9%	-	-	42.1%	-	-	-	-	69.1%	-	-	-	-	-	-	-	-	8.2%
Health Care	12.6%	18.1%	-5.2%	-	-	-17.8%	-	-	-	-	-	-	-24.7%	14.4%	-	-	-17.8%	1.0%
Industrials	11.6%	-20.2%	0.0%	11.2%	12.5%	-	-	-	-	-	-	18.1%	-	-	-	-	-	1.7%
IT	-	-	-2.2%	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-0.4%
LPT	-	-	6.8%	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.2%
Materials	-	-	-	18.5%	-	-	-	-	-	-	-	-	-	-	-	-	-	1.1%
Telecommunication	-	-	-2.3%	-	-5.0%	-	-6.5%	-	-	-	-	-	-	-	-	-	-	-1.3%
Utilities	-7.4%	-	-5.6%	8.1%	-6.2%	-	-	-	-	-	-	-	-	-	-	-	-	-1.6%
Value	-	-	-6.4%	14.9%	-43.6%	-21.5%	-	-42.6%	-49.4%	-	-	-	-	33.7%	-	52.7%	-21.5%	-3.9%
Growth	-	-	-	-	-	53.8%	-	-	-	-	-	-	-	-	-	-52.9%	53.8%	-1.1%
ASX ex-150	-	-	-	-	-	-	-	-	-	-	81.8%	47.0%	109.4%	-	-	-	-	2.8%
ASX Small Ordinaries	-	-	-	-	-	-	-	23.2%	-	-	78.3%	81.5%	115.2%	-	-9.0%	-	-	4.2%
Model Adjusted R^2	93.3%	88.4%	98.8%	93.8%	94.7%	90.2%	94.6%	89.3%	91.3%	87.6%	78.5%	88.5%	83.4%	94.3%	94.6%	85.2%	90.2%	N/A
Model Standard Deviation	0.8%	1.3%	0.3%	0.8%	0.8%	1.2%	0.7%	1.1%	1.2%	1.5%	1.7%	1.4%	2.1%	0.8%	0.7%	1.2%	1.2%	N/A
Residual Skew	-1.2	-0.5	0.3	-0.2	0.6	0.1	-0.6	-0.1	0.2	-0.2	-0.3	-0.3	0.3	0.1	0.7	-0.4	0.1	N/A
Residual Kurtosis-3	4.3	-0.1	0.8	0.6	1.2	0.3	1.9	-0.8	0.6	-1.0	0.2	0.4	0.5	-0.2	1.8	-0.3	0.3	N/A
Residual DW Statistic-2	0.1	-0.1	0.1	0.1	-0.2	-0.2	-0.0	-0.2	0.6	0.2	-0.3	-0.4	-0.1	0.1	0.4	-0.1	-0.2	N/A
Residual Autocorrelation	-8.0%	0.8%	-3.5%	-5.4%	7.8%	8.2%	-0.3%	6.6%	-29.6%	-23.9%	13.9%	19.9%	5.1%	-13.4%	-22.5%	6.1%	8.2%	N/A

Note: *All indexes represent exposures after removal of the effects of all factors ranked higher in the list, as described in 4.2.*

Detailed below are illustrative comments of the regression results for one of the Fund's Australian equity managers (viz. manager Q).

Manager Q

Manager Q adopts a quantitative investment process, which combines seven key factors. The manager's portfolio is structured to be broadly style neutral over the long-term, although biases to value or growth may be evident over shorter time periods subject to prevailing market conditions. The table below displays the key regression results for the manager.

Table 2: Australian Equity Managers – Manager Q's regression results

Regression Factor	Coefficients	Standard Error	t-Statistic	p-Value	Lower 95%	Upper 95%
Manager's ex-post Alpha (p.a.)	1.6%	2.8%	0.57	57.19%	-4.0%	7.1%
ASX300A index	113%	5.0%	22.40	0.00%	103%	123%
ASX Health Care Sector ^	-18%	6.3%	2.85	0.63%	-30%	-5%
ASX Value Index ^	-21%	10.3%	2.08	4.19%	-42%	-1%
ASX Growth Index ^	54%	24.4%	2.21	3.17%	5%	103%

Note: *Factors marked ^ represent factor exposures after removal of the effects of all factors ranked higher in the list.*

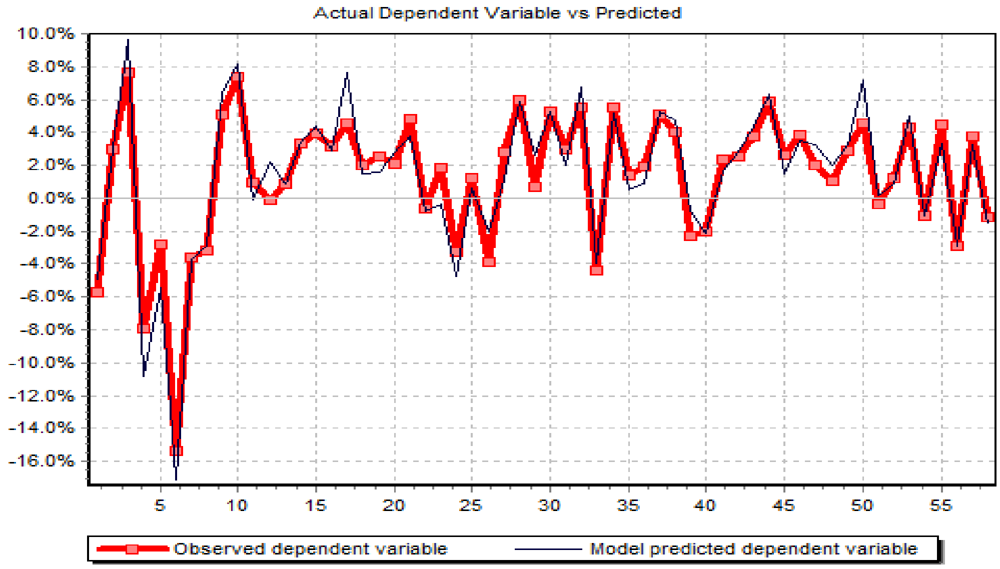
Model Fit		
Model Adjusted R^2	90.1%	
Model Standard Deviation	1.3%	
F-Significance	133.12	Significant at the 99.95% level
Residual Analysis		
Residual Skew	0.1	
Residual Kurtosis-3	0.3	
Residual Auto-Correlation	8%	

TURBOs assessed that the manager's returns could be described as generating an alpha return of 1.6% p.a. plus 113% of the ASX 300 Accumulation Index, with a negative Health Care exposure, and a solid growth bias. All factors, other than the manager's *ex-post* alpha are highly significant. The overall model fit is good, with a 90.1% adjusted R^2 and an extremely high F-statistic. In addition, the residuals approximate a normal distribution with almost no evidence of skewness or kurtosis and only moderate auto-correlation.

The large exposure to the ASX 300 Accumulation Index is somewhat higher than expected, given that the manager has constructed their portfolio to be broadly beta neutral. The negative exposure to the Health Care sector is considered largely a period specific outcome and not an inherent bias in the manager's investment process, and may also be the contra position to the higher ASX 300 beta (i.e. TURBOs is viewing the manager as higher market beta, but below market exposure to Health Care, which is a high beta sector).

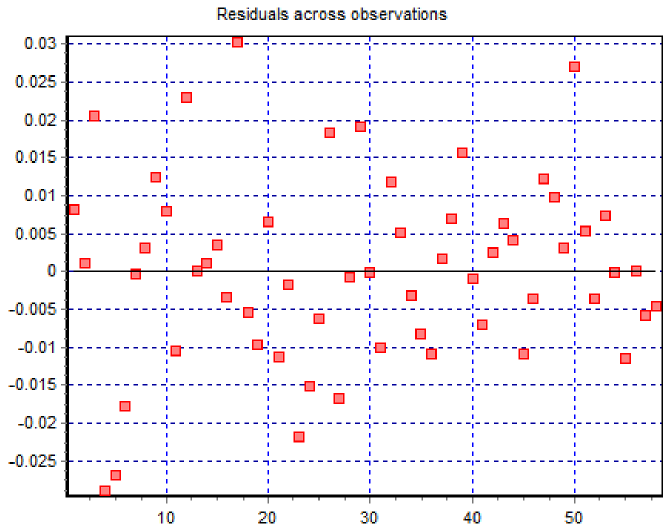
TURBOs also displays the results of the fit graphically, as shown in the following charts. The residuals of the regression fits for each manager is carefully analysed by UniSuper, as these provide insight as to the higher moments of each manager's alpha distribution.

Chart 2: Contrasting manager Q's observed returns with the best fit obtained



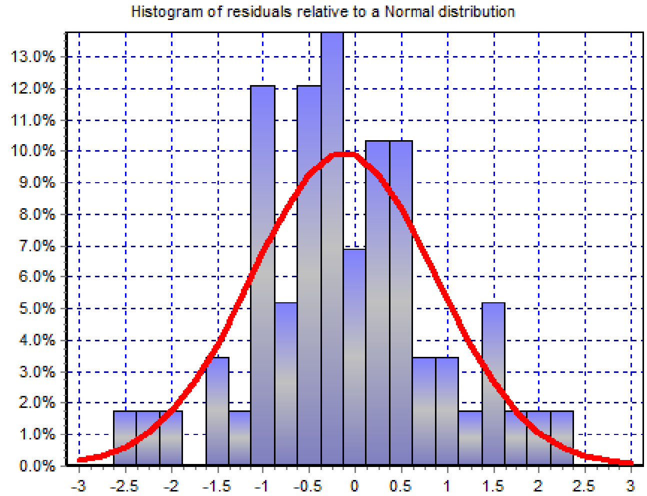
The chart above contrasts the modelled factors (given by the thin black line) against the manager's actual returns (given by the red curve), demonstrating that a good fit was attained. The next chart provides a scatter plot of the error terms (or residuals), which effectively represents the distribution of the manager's alpha.

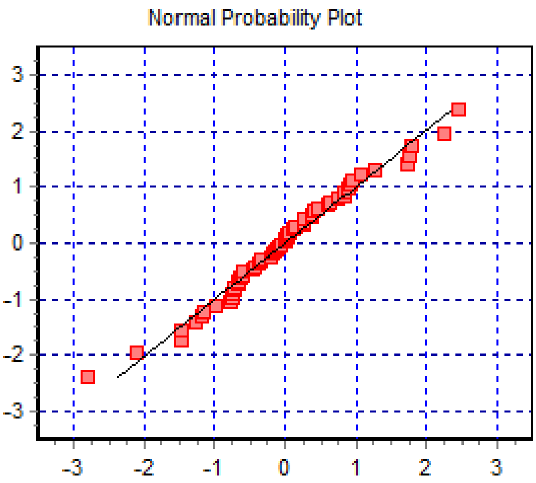
Chart 3: Scatter plot of error terms for Manager Q



The scatter plot demonstrates that the residuals are distributed in a broadly random manner around a mean of zero. Another manner of testing the goodness of fit, is to consider a histogram of residuals relative to a normal distribution, as well as a normal probability plot, as shown in the next chart.

Chart 4: Manager Q's residual distribution





The above histogram of residuals and normal probability plot are reasonably closely aligned to a normal distribution (given by the red line and the 45^0 line for the normal probability plot).

3.3 Expected vs. Actual Beta Exposure by Accumulation Option

Once TURBOs completes the analysis presented in 3.1 for each manager and investment (some of the private equity, infrastructure and direct property manager return series are grouped together due to the high incidence of stale prices), TURBOs assesses the extent to which each Beta factor is correlated to the Fund's benchmarks. The same multiple regression techniques are used to map each factor to the Fund's benchmarks, as was used to review each of the Fund's managers. By aggregating the results for each Accumulation Option one can contrast the Option's derived beta exposure to that expected from each Option's SAA, as shown in the table below:

Table 3: Expected vs. Actual Beta Exposure For Each Option

Beta Attribution	**High Growth (%)**		**Growth Option (%)**		**Balanced Option (%)**		**Cons. Balanced (%)**		**Capital Stable (%)**	
	SAA	**Fitted Beta Factors**	**SAA**	**Fitted Beta Factors**	**SAA**	**Fitted Beta Factors**	**SAA**	**Fitted Beta Factors**	**SAA**	**Fitted Beta Factors**
Australian Listed and Private equity	41.8	38.4	33.7	31.3	28.3	26.6	21.0	20.5	11.0	11.0
International Listed and Private Equity	37.7	35.4	33.8	32.5	27.3	26.9	19.0	20.0	9.0	9.5
Domestic and International Bonds	0.0	4.2	7.5	10.6	25.0	26.8	40.0	39.8	50.0	50.0
Direct Property	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0
Index Linked Bonds	0.0	-2.9	7.5	5.5	5.0	3.4	5.0	4.4	7.5	6.7
Infrastructure	10.5	10.5	7.5	7.5	4.5	4.5	0.0	0.0	0.0	0.0
Listed Property Trusts	3.0	3.5	3.0	3.3	3.0	3.1	3.0	2.9	3.0	2.8
Cash	0.0	-1.7	0.0	3.9	0.0	3.9	5.0	8.7	12.5	16.9
Extraneous risk premia	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	**100.0**	**94.5**	**100.0**	**101.7**	**100.0**	**102.2**	**100.0**	**103.4**	**100.0**	**103.9**

The above table assesses the extent to which each Option's beta exposure differs to the Fund's SAA Benchmarks. For example, the Balanced Option has a slightly lower equity and index Linked Bond beta than expected, which is offset by a higher than expected fixed interest and cash allocation. Overall the observed beta exposure for each option is broadly in line with the option's SAA.

3.4 MARGINAL AND PROPORTIONAL CONTRIBUTION TO RISK

Having assessed each manager's *ex-post* excess returns, TURBOs computes the marginal and proportional contribution to total Option risk from each manager/investment. For the Balanced Option (which represents UniSuper's default option, and hence is the most popular option), the *ex-post* total Option volatility amounted to 8.6% p.a. over the three years to June 2008. The total Option risk was below the SAA long-term assumptions for the Balanced Option (9.8%), primarily as equities exhibited unusually low volatility until mid-2007. Clearly, the authors expect the Option's 3-year rolling observed volatility to increase over the coming months.

By considering the covariance matrix of each manager's *ex-post* alpha returns, TURBOs is able to derive the Option's tracking error, which amounted to 1.1% for the Balanced Option. The tables below demonstrate how each asset class contributed to the observed Option risk and tracking error, for the Balanced Option.

Table 4: Balanced Option -Proportional Contribution to Risk and Tracking Error

Asset Class	Balanced Option		
	Asset Class Weight (%)	Proportional Contribution to Option Volatility (%)	Proportional Contribution to Tracking Error (%)
Australian Equities	**27.5**	**45.1**	**21.6**
Enhanced Passive	4.8	2.7	0.6
Growth	3.9	5.9	5.5
Long/Short	1.5	3.0	1.2
Neutral	10.8	16.1	5.7
Small Caps	1.0	8.5	5.5
Value	5.6	8.8	3.2
International Equities	**25.0**	**34.7**	**32.6**
Direct Property	**7.0**	**-0.1**	**5.0**
Listed Property	**3.0**	**15.8**	**-0.6**
Infrastructure	**4.5**	**1.0**	**32.5**
Private Equity	**3.0**	**3.5**	**7.8**
TOTAL GROWTH ASSETS	**70.0**	**100.0**	**99.0**
Domestic Fixed Interest	**17.5**	**-0.1**	**0.4**
International Fixed Interest	**7.5**	**0.1**	**1.0**
Domestic Indexed Bonds	**5.0**	**0.0**	**-0.3**
Cash	**0.0**	**0.0**	**0.0**
TOTAL DEFENSIVE ASSETS	**30.0**	**0.0**	**1.0**
Option Volatility & Tracking Error		**8.64**	**1.11**

The above tables demonstrate that most of each Option's total volatility stems from the listed equity portfolios. Not surprisingly, direct property and defensive assets reduced the Option's risk.

When considering tracking error, the authors note that the alternative assets (in particular infrastructure assets) generate most of the option's tracking error – however this result partly arises as UniSuper is still in the process of obtaining appropriate risk factors for these asset classes. This shortcoming generates an overly low beta, and high alpha contribution to returns, with an excessive tracking error for alternative assets.

In addition to the asset class contribution to total risk, TURBOs computes the marginal and proportion risk from each manager. The table below summarises the top 20 managers' contribution to total risk and tracking error for the Balanced Option.

Table 5: Balanced Option – Top 20 Contributors to total risk

Rank	Asset Sub Class	Manager	Option's Weight in Manager (%)	Marginal Contribution to Risk (%)	Proportional Contribution to Risk (%)
1	Listed Properties	LPT Manager A	1.4	2.8	8.9
2	Listed Properties	LPT Manager B	1.6	2.1	6.6
3	Asia Ex-Japan	IEQ Manager A	0.7	1.1	3.6
4	Asia Ex-Japan	IEQ Manager B	0.6	1.1	3.6
5	Australian Equities	AEQ Manager D	2.0	1.0	3.3
6	Australian Equities	AEQ Manager A	2.5	1.0	3.3
7	Value	IEQ Manager C	1.9	1.0	3.3
8	Australian Equities	AEQ Manager I	0.9	1.0	3.2
9	Australian Equities	AEQ Manager L	0.4	1.0	3.2
10	Australian Equities	AEQ Manager J	2.1	1.0	3.2
11	Australian Equities	AEQ Manager P	2.5	1.0	3.1
12	Australian Equities	AEQ Manager N	2.0	0.9	3.1
13	Australian Equities	AEQ Manager Q	1.7	0.9	3.0
14	Australian Equities	AEQ Manager K	0.2	0.9	2.9
15	Emerging Market	IEQ Manager D	1.8	0.9	2.9
16	Growth	IEQ Manager E	1.8	0.9	2.8
17	Growth	IEQ Manager F	1.8	0.9	2.8
18	Australian Equities	AEQ Manager C	5.6	0.9	2.7
19	Australian Equities	AEQ Manager G	2.3	0.8	2.7
20	Australian Equities	AEQ Manager B	2.1	0.8	2.7
	Total		**36.0**	**22.0**	**70.7**

Although the twenty managers tabulated above account for 36% of the Balanced Option's assets, they account for 71% of the Option's total risk. As expected, the top 20 managers/investments reside within the listed equity asset classes. Overall, the authors conclude the Balanced Option is well diversified – with the possible exception of the listed property mandates, which account for 15.5% of the Option's total risk, but only 3% of the assets. The recent extreme volatility exhibited by the listed property sector has affected the contribution to Option risk from the Listed Property Trust (LPT) managers.

One possible finding is that an additional manager (possibly with a low tracking error mandate) could be introduced to the LPT manager line-up to help reduce the asset class's risk.

The next step in the Fund's risk budgeting analysis is to consider the marginal contribution to tracking error. The top 20 contributors are presented on the table overleaf.

Table 6: Balanced Option – Top 20 Contributors to tracking error

Rank	Asset Sub Class	Manager	Option's Weight in Manager	Marginal Contribution to Risk	Proportional Contribution to Risk
			(%)	(%)	(%)
1	Infrastructure	Diversified	0.7	0.064	12.9
2	Infrastructure	Airports	3.0	0.053	10.6
3	Private Equity	Buy Outs	0.5	0.033	6.7
4	Infrastructure	Utility	2.3	0.033	6.6
5	Value	IEQ Manager A	1.9	0.027	5.4
6	Long/Short	IEQ Manager B	1.8	0.023	4.6
7	Direct	Direct Property	7.0	0.022	4.4
8	Australian Equity	AEQ Manager K	0.5	0.021	4.3
9	Value	IEQ Manager C	1.8	0.021	4.3
10	Long/Short	IEQ Manager D	2.0	0.019	3.9
11	Enhanced Passive	IEQ Manager E	6.8	0.019	3.9
12	Infrastructure	Roads	1.5	0.019	3.9
13	Growth	IEQ Manager F	1.8	0.018	3.6
14	Enhanced Passive	IEQ Manager G	6.9	0.017	3.3
15	Private Equity	Venture Capital	0.1	0.015	3.1
16	Asia Ex-Japan	IEQ Manager H	0.7	0.015	3.0
17	Australian Equity	AEQ Manager A	2.5	0.014	2.8
18	Australian Equity	AEQ Manager B	2.1	0.012	2.4
19	Growth	IEQ Manager I	1.8	0.011	2.2
20	Australian Equity	AEQ Manager D	2.0	0.010	2.0
	Total		**47.8**	**0.46**	**93.9**

Tracking error is computed by considering the standard deviation of each manager's excess returns. When considering tracking error, care is required in the interpretation of the results for the alternative asset classes, due to the stale, smoothed and serially correlated nature of their return stream. Nonetheless, the above table demonstrates that the bulk of the Balanced Option's tracking error results from alternative assets. These findings were expected, UniSuper selects alternative assets based on their risk-return characteristics, and these assets offer favourable risk adjusted returns.

3.5 DERIVATION OF EACH OPTION'S EX-ANTE ALPHA

The next step in the risk budgeting process that TURBOs computes, is the estimate of each manager's *ex-ante* (or forecast) alpha. The authors utilised an adaptation of the Black-Litterman Model (BLM) to derive the estimates presented in the table overleaf:

Table 7: ex-Ante Alpha Forecasts for the Balanced Option

Asset sub-class	Manager	Balanced Option Manager Weights (%)	Observed ex-Post Alpha (%)	Tracking Error (%)	UniSuper's View of Each Manager's ex-Ante Alpha (%)	Confidence (That Alpha Lies Within 1% of View) (%)	TURBOs ex-Ante Alpha (%)
Australian Equities *		**27.5**	**0.2**	**1.4**	**1.4**		**1.3**
	AEQ Manager A	2.1	-0.8	4.3	1.3	70	1.3
	AEQ Manager B	1.8	3.2	4.7	1.2	80	1.3
	AEQ Manager C	4.8	1.0	1.5	1.1	85	1.0
	AEQ Manager D	1.7	-2.6	3.9	1.2	65	1.2
	AEQ Manager E	2.6	3.1	4.5	1.8	75	1.9
	AEQ Manager G	2.0	0.8	2.5	0.9	75	0.9
	AEQ Manager H	1.9	3.2	5.2	1.8	60	1.8
	AEQ Manager I	0.8	0.0	5.3	0.7	55	0.5
	AEQ Manager J	1.7	2.6	5.3	1.5	60	1.5
	AEQ Manager K	0.5	2.4	8.9	2.6	60	2.4
	AEQ Manager L	0.4	1.3	7.9	2.2	70	1.6
	AEQ Manager M	0.2	-6.0	12.8	2.4	55	2.1
	AEQ Manager N	1.7	-3.3	4.4	1.1	55	0.9
	AEQ Manager O	1.9	-1.0	3.8	0.9	85	0.9
	AEQ Manager P	2.1	-7.3	6.8	1.1	70	1.1
	AEQ Manager Q	1.5	1.7	5.8	2.5	65	2.5
International Equities		**25.0**	**-2.8**	**3.2**	**1.5**		**1.5**
Direct Property		**7.0**	**4.4**	**3.1**	**0.0**		**0.0**
Listed Property		**3.0**	**-7.3**	**6.2**	**0.2**		**0.1**
Infrastructure		**4.5**	**9.7**	**13.9**	**0.0**		**0.0**
Private Equity		**3.0**	**3.2**	**5.7**	**0.0**		**0.2**
Domestic Fixed interest		**17.5**	**-0.7**	**0.1**	**0.2**		**0.3**
International Fixed Interest (Hedged)		**7.5**	**-0.1**	**0.9**	**0.8**		**0.7**
Index Linked Bonds		**5.0**	**-3.4**	**0.6**	**0.0**		**0.0**
Total/ Weighted Average		**100.0**	**-0.3**	**1.1**	**0.9**		**0.9**

** There was insufficient historic return data for Australian Equity Manager F, for this analysis as at the date of the investigation. In addition, the TURBOs ex-ante alphas generated are sensitive to the value of tau.*

The above table combines the investor's view with the each manager's *ex-post* alpha to assess the *ex-ante* alpha for that manager. TURBOs calculates the expected *ex-ante* alpha for each manager, weights the results and generates an overall *ex-ante* alpha estimate of 0.9% for the Balanced Option. Similar calculations were performed for the High Growth Option (*ex ante* alpha estimate amounts to 1.0%); Growth Option (0.9%); Conservative Balanced Option (0.7%); and the Capital Stable Option (0.5%);

3.6 ASSESSING WHETHER ACTIVE MANAGEMENT IS EXPECTED TO ADD VALUE

Having derived an Option level *ex-ante* alpha estimate, TURBOs is then able to assess whether each Option is expected to generate sufficient alpha, to justify a departure from a passive replication of the Fund's SAA Benchmarks. Hurdles for each option are derived in section 7 of Appendix 1. In essence, the ratio of the Option's ex-ante alpha divided by the increase in volatility relative to the benchmark must exceed the derivate of the Fund's constrained efficient frontier, to justify the use of active management.

The derivative of the Fund's constrained efficient frontier, using the Fund's normative long-term assumptions at the Balanced Option's volatility level amounts to 0.28. If the Fund had invested passively and precisely matched its benchmarks, then the Option would have generated a volatility of 7.1%. TURBOs estimated that the Option's total volatility amounted to 8.6%. Hence, to justify an active program the *ex-ante* alpha divided by the increase in volatility (of 1.6%) must exceed 0.4% for the Balanced Option. The results for all non-SRI diversified options are presented in the table below:

Table 8: Required ex-Ante alpha to Justify a Departure From the Balanced Option's SAA

Option	**High Growth**	**Growth**	**Balanced**	**Cons. Balanced**	**Capital Stable**
Derivative of the constrained efficient frontier	0.14	0.26	0.28	0.34	0.45
Actual volatility had the fund tracked benchmark	10.4%	8.7%	7.1%	5.0%	2.8%
Observed total volatility	11.7%	10.2%	8.6%	6.8%	4.1%
Observed increase in risk, as a result of active management and strategic tilting	1.4%	1.5%	1.6%	1.8%	1.3%
Minimum required *ex-ante* alpha (net of fees)	**0.2%**	**0.4%**	**0.4%**	**0.6%**	**0.6%**

Hence to justify the use of deviating from a passive investment philosophy, the Fund has to exceed a minimum excess return of 0.2% for the High Growth Option, 0.4% for the Growth Option, 0.4% for the Balanced Option, 0.6% for the Conservative Balanced Option and 0.6% for the Capital Stable Option.

By combining the findings from Table 1 (the expected vs. observed beta from each option) with table 5 (the *ex-ante* alpha forecasts for each manager) and table 6 (the minimum alpha hurdle for each manager), the investor is able to assess the extent to which each option's benchmarks are expected to be met, and whether the Fund expects to exceed the minimum hurdle required to justify active management. The results of this analysis are provided in the table overleaf.

Table 9: Risk Budgeting and Factor Analysis – Summary Findings

Return Attribution	High Growth		Growth Option		Balanced Option		Conservative Balanced		Capital Stable	
	SAA (%)	Observed Beta Factors (%)	SAA (%)	Observed Beta Factors (%)	SAA (%)	Observed Beta Factors (%)	SAA (%)	Observed Beta Factors (%)	SAA (%)	Observed Beta Factors (%)
Australian Listed and Private equity	41.8	38.4	33.7	31.3	28.3	26.6	21.0	20.5	11.0	11.0
International Listed and Private Equity	37.7	35.4	33.8	32.5	27.3	26.9	19.0	20.0	9.0	9.5
Domestic and International Bonds	0.0	4.2	7.5	10.6	25.0	26.8	40.0	39.8	50.0	50.0
Direct Property	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0
Indexed Linked Bonds	0.0	-2.9	7.5	5.5	5.0	3.4	5.0	4.4	7.5	6.7
Infrastructure	10.5	10.5	7.5	7.5	4.5	4.5	0.0	0.0	0.0	0.0
Listed Property Trusts	3.0	3.5	3.0	3.3	3.0	3.1	3.0	2.9	3.0	2.8
Cash	0.0	-1.7	0.0	3.9	0.0	3.9	5.0	8.7	12.5	16.9
Extraneous risk premia	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	**100.0**	**94.5**	**100.0**	**101.7**	**100.0**	**102.2**	**100.0**	**103.4**	**100.0**	**103.9**
Expected return from Beta sources	8.3	7.9	7.8	7.7	7.3	7.3	6.6	6.7	5.8	6.0
Impact of rebalancing	0.9	0.8	0.7	0.7	0.5	0.5	0.3	0.3	0.1	0.1
Ex-Ante alpha		1.0		0.9		0.9		0.7		0.5
Total expected return	**9.2**	**9.7**	**8.4**	**9.3**	**7.7**	**8.6**	**6.9**	**7.8**	**5.9**	**6.6**
Expected excess return		0.55		0.88		0.88		0.90		0.67
Minimum hurdle to justify active management		0.19		0.38		0.44		0.61		0.57

Notes: ^ The observed beta factors for direct property and infrastructure have been set to the benchmark, and the asset classes ex-ante forecast has been set to zero.

The above table demonstrates that the Fund's current manager line-up for each of the non-SRI diversified options generates a beta exposure that is broadly in line with the option's SAA, and that each option's ex-ante alpha exceeds the minimum hurdle required to justify active management.

3.7 REVERSE OPTIMISATION

The final calculation performed by TURBOs is an assessment of an optimal manager line-up (optimal in the sense of generating the greatest likelihood of beating the required alpha). This is a theoretical construct and considerable care is required in interpreting the results. The table below summarises the optimised portfolio generated from TURBOs for the Balanced Option:

Table 10: Reverse Optimisation - Balance Option

Asset sub-class	Manager	Current Manager Line-up			Results of Reverse-Optimisation		
		Manager Weights (%)	Tracking error (%)	*ex-Ante* alpha (%)	Revised Weight (%)	Tracking Error (%)	*ex-Ante* Alpha (%)
Australian Equities *		**27.5**	**1.4**	**1.3**	**27.5**	**0.9**	**1.2**
	Manager A	2.1	4.3	1.3	0.0		
	Manager B	1.8	4.7	1.3	0.2		
	Manager C	4.8	1.5	1.0	16.2		
	Manager D	1.7	3.9	1.2	2.8		
	Manager E	2.6	4.5	1.9	0.3		
	Manager G	2.0	2.5	0.9	1.0		
	Manager H	1.9	5.2	1.8	1.0		
	Manager I	0.8	5.3	0.5	0.0		
	Manager J	1.7	5.3	1.5	1.7		
	Manager K	0.5	8.9	2.4	2.0		
	Manager L	0.4	7.9	1.6	0.0		
	Manager M	0.2	12.8	2.1	0.0		
	Manager N	1.7	4.4	0.9	2.3		
	Manager O	1.9	3.8	0.9	0.0		
	Manager P	2.1	6.8	1.1	0.0		
	Manager Q	1.5	5.8	2.5	0.0		
International Equities		**25.0**	**3.2**	**1.5**	**25.0**	**3.8**	**1.8**
Direct Property		**7.0**	**4.4**	**0.0**	**7.0**	**4.4**	**0.0**
Listed Property		**3.0**	**6.2**	**0.2**	**3.0**	**6.6**	**0.2**
Infrastructure		**4.5**	**13.9**	**0.0**	**4.5**	**13.9**	**0.0**
Private Equity		**3.0**	**5.7**	**0.0**	**3.0**	**5.5**	**0.2**
Domestic Fixed interest		**17.5**	**0.1**	**0.2**	**17.5**	**0.0**	**0.3**
International Fixed Interest		**7.5**	**0.9**	**0.8**	**7.5**	**1.3**	**0.9**
Index Linked Bonds		**5.0**	**0.6**	**0.0**	**5.0**	**0.6**	**0.0**
Total/ Weighted Average		**100.0**	**1.14**	**0.85**	**100.0**	**1.22**	**0.91**
Information Ratio				**0.749**			**0.750**

** There was insufficient historic return data for Australian Equity Manager F, for this analysis as at the date of the investigation.*

As can be seen in the above table, TURBOs recommends that greater emphasis be placed on high information ratio managers (e.g. enhanced passive mandates). By utilising the recommended manager weights, the Balanced Option is expected to obtain a slightly higher information ratio. However, the change in information ratio is slight, suggesting that the Option's current manager line-up has been well considered.

4 PRACTICAL CONSIDERATIONS

4.1 Calibrating the BLM

The BLM has been discussed in a variety of sources (see for example Black & Litterman (1992), He & Litterman (2002), Litterman (2003a), Walters (2008) and Meucci (2008)). The BLM is discussed and adapted to consider active risk in section A1.4.2. Whilst the development of the model and the investors' views is reasonably straightforward, care is required in the calibration of tau as well as the formulation of omega matrix.

4.1.1 Calibrating tau

Walters (2008) explains that it is common for users of the BLM to be confused as to the appropriate value for tau. He and Litterman (2002) use a value of 0.025, whereas Satchell and Scowcroft remark that many people use a value of τ close to 1. Several other authors (eg. Meucci 2008) completely eliminate τ.

Personal discussions with the author of the BLM (Litterman, 2008) suggest that τ should be such that the standard deviations are of a similar scale to that of $\overline{\Pi}$. Litterman recommends a value around 0.3 when one is considering total returns. The output from the BL model is not overly sensitive to the selected value of τ. Our view is that a higher value of τ (viz. in the 0.7-0.9 range) generates a more stable and interpretable *ex-ante* alpha estimate for the ex-ante alpha estimates.

4.1.2 Calculating Omega

The Omega matrix represents one's confidence in each manager or stock's ability to generate alpha. Specifically, Omega represents the variance of the view matrix *Q*.

Walters (2008) provides a range of methods that can be used to derive the omega matrix. The two methods that most appealed to us were the use of confidence intervals and using the variance of the residuals from the factor models. We selected the confidence interval approach. We did this by defining a confidence interval that each manager or stock would outperform their benchmark within a 1% range. Walters (2008) provides the following example of the method we use: "*Asset 2 has an estimated 3% mean return with the expectation that it is 67% likely to be within the interval (2.5%,3.5%). Knowing that 67% of the normal distribution falls within 1 standard deviation of the mean, allows us to translate this into a variance of $(0.005)^2$*". We compute the associated variance of each manager's outperformance, and these values form the diagonal of the Omega matrix.

4.2 Handling collinearity of risk factors

Given the high levels of collinearity between factors (for example the return for large cap Australian stock index (ASX 100) is highly correlated with the return for ASX 300 stocks), we adopt a standard econometric technique of creating new factor return series, which equate to the residuals of the given factor, after regressing on all prior factors. The structure is sequential and requires the selection of an ordering of the factors. In most cases, we choose what we consider to be the primary market indicator as the first factor and then order the remaining factors to reflect the significance that we expect the factor returns to have in explaining the returns of managers/investments in each sector. For example, for the Australian shares sector we chose the ASX300 as the primary factor and calculate, say, the residual returns of the Value index returns, after regressing on the ASX300 index returns. Note that this provides both an estimate of the market beta of the Value index to the ASX300 market and of the average excess return of the Value return over the ASX300 for the estimation period (an "alpha" estimate) and the residual series which can be interpreted as the return series for value after adjust for its market component. Lower order factors are regressed on the primary factor and each of the prior residual returns series from the higher order regressions. In the notation below, we write these residuals as $F^{*}_{k,t}$, and for ease of expression we can write $F^{*}_{1,t} = F_{1,t}$ and refer to them as the residual factors.

Note that each resulting risk factor is simply the residual of the excess return of factor *i* after allowing for the exposure to the other (higher order) residual factors, and these residuals are orthogonal. This approach also has the advantage of providing some insight into the nature of the markets for the period on which the estimates are based.

4.3 Managing alternative assets

Alternative assets are often valued with reference to a mathematical model, have stale and infrequently quoted prices and as such display a smoothed, serially correlated return stream. Contrasting their returns to listed indices is of limited value. To overcome these concerns, we contrast the returns from these asset classes to the rolling geometric average returns from listed markets over 2 years[(2)].

4.4 Interpreting the derivative of the constrained efficient frontier

The derivative of the constrained efficient frontier represents the hurdle that must be achieved to justify using active management, rather than altering the beta allocation for a given strategic asset allocation. Three practical consequences arise from this simple observation:

1. The greater the slope of the constrained efficient frontier, the greater the hurdle. As such, it is more likely that the hurdle would be achieved for higher risk strategies (such as for a High Growth Option) than for a Cash Option or other conservative options (where the slope of the constrained efficient frontier is at its maximum). For these more defensive strategies it may well be more appropriate to increase beta risk (e.g. by increasing duration or bonds or by introducing credit into portfolios) rather than to increase the use of active risk or port alpha from other sources.

2. Strategies that reduce overall option risk, but with the cost of a slight reduction in return, are more appropriate within lower risk Options, with a higher slope of the constrained efficient frontier. As an example, currency hedging of international growth assets generally reduces Option volatility, but at the cost of implementing the hedge. It may thus be appropriate to maintain a higher currency hedge ratio for international growth assets within conservative Options than for an Option with a higher risk tolerance.

3. The greater the number of constraints imposed by the investor, the flatter the slope of the constrained efficient frontier, and adopting active management is more easily justified.

4.5 Reverse optimisation

The optimiser favours managers and stocks with lower tracking errors to benchmark and implied strong performance compared to risk. However, the optimiser does not consider a range of other factors that are relevant to the structuring portfolio, including the capacity of the manager and the importance of diversifying manager risk. Whilst the TURBOs optimised portfolio provides some insights into potential portfolio structure, it is only a tool and does not take into account a range of other (predominantly qualitative) portfolio construction considerations, including:

- The type of manager (i.e. 'developing' or mature, boutique or institutional);
- Capacity of the manager;
- Desired style objective of the overall portfolio;
- Manager-specific operational risks and the need to diversify manager exposure;
- The overall bias to small caps (and the smaller end of the small caps market); and
- Other qualitative considerations.

TURBOs is particularly helpful in reviewing exposures across managers and in identifying and assessing the key sources of return for a manager, as opposed to being used to construct a final portfolio. In this regard, optimisers tend to have difficulty producing meaningful portfolios, given the difficulty in incorporating a large number of variables (including qualitative components) and the tendency to concentrate allocations to managers with slightly better performance characteristics.

5 CONCLUSIONS

Risk budgeting is the process of setting a target level of risk to be accepted at the portfolio level, and allocating this risk across a number of investments in the most efficient manner in order to maximise returns whilst containing risk within the agreed targets.

UniSuper has developed an in-house risk budgeting and factor analysis program that monitors the extent to which the Fund deviates from its Strategic Asset Allocation. TURBOs provides the Fund with a formal framework for discussion and analysis. The resulting analysis provides insights that help formulae the Fund's investment arrangements.

Appendix 1 - Mathematical Formulation

To aid the reader, a glossary of notation is provided in Appendix 2.

A1.1 Background

Let W_m^o denote the Strategic Asset Allocation (SAA) weight for Option *o* within asset class *m.*

Let $w_{i,t}^{a_i}$ denote the weight (as a proportion of the total Fund) for manager *i* who operates in asset class a_i at time *t*. Hence $\sum_{i=1}^{N} w_{i,t}^{a_i} = 1$. Where *N* denotes the total number of managers spanning all asset classes.

Let $w_{i,t}^{o,a_i}$ denote manager *i*'s weight within Option *o* (manager *i* invests in asset class a_i) at time *t.*

Then $w_{i,t}^{o,a_i} = w_{i,t}^{a_i} W_m^o$ ...(1), where *m* maps onto asset class a_i for the weight W_m^o.

Let $\overline{w}_t$ denote the vector of weights of manager holdings at time *t* within Option *o.*

Hence $\overline{w}_t = \begin{bmatrix} w_{1,t}^{a_1} W_{a_1}^o \\ \ldots \\ w_{N,t}^{a_N} W_{a_N}^o \end{bmatrix}$ ...(1.1)

Further: $\overline{w}_t{}'\overline{1} = 1$, where $\overline{1}$ denotes the unit vector $\begin{bmatrix} 1 \\ \ldots \\ 1 \end{bmatrix}$.

For ease of notation vectors and matrices will exclude the time *(t)* and option (*o*) suffix in this appendix, although each matrix is assumed to be time and option dependent.

If $\hat{\mu}_{i,t}^{a_i}$ denotes the ex-ante expected return from manager *i* at time *t* in asset class a_i, while $\mu_{i,t}^{a_i}$ denotes the observed return from manager *i* at time *t*. Then $\hat{\overline{\mu}}$ is a vector of *ex-ante* expected returns for each manager, at time, and $\hat{\overline{\mu}} = \begin{bmatrix} \hat{\mu}_{1,t}^{1} \\ \ldots \\ \hat{\mu}_{N,t}^{M} \end{bmatrix}$ …(1.2)

Further, $\overline{w}'\hat{\overline{\mu}}$ equals the Fund's expected return in a year's time (on the basis that the manager weights remain constant over the timeframe considered).

The value of $\hat{\overline{\mu}}$ is derived in sections A1.2 and A1.4 below.

A1.2. Estimating Each Manager's Factor Exposures

Each manager's return can be considered as the sum of a beta (or market) factor exposure together with an active manager return (alpha). The first step in estimating the value of $\hat{\mu}_{i,t}^{a_i}$ is to determine each manager's factor exposure or beta exposures. The more difficult step in the process is to estimate each manager's prospective *ex-ante* alpha. This is discussed in section 4.

A1.2.1. Decomposing Manager Returns

Sharpe (1964) derived the Capital Asset Pricing Model (CAPM), which states that a share's expected performance at time *t* (given by $E[R_{s,t}]$) is dependent on the extent to which the share is correlated to the market (referred to as the share's systemic risk or beta). Specifically, Sharpe derived the CAPM formula:

$$E[R_{s,t}] = R_{f,t} + \hat{\beta}(E[R_{M,t}] - R_{f,t}) + \tilde{\varepsilon}_{s,t} \quad \text{.... (2.1.1)}$$

Where $\hat{\beta} = \dfrac{\hat{\sigma}_{s,t}^{M}}{Var(\hat{R}_M)}$ and $\hat{\sigma}_{s,t}^{M}$ denotes the estimate covariance between the security (*s*) and an appropriate market index (*M*) at time *t,* while $Var(\hat{R}_M)$ denotes the estimated variance of the market at time *t* and $\tilde{\varepsilon}_{s,t}$ represents the error term.

CAPM was extended by Ross (1976), in his formulation of Arbitrage Pricing Theory (APT). APT states that the expected return of a financial asset can be modelled as a linear function of various macro-economic factors $\{F_k\}$ or theoretical market indices, where sensitivity to changes in each factor is represented by a factor-specific beta coefficient ($\beta_{s,k,t}$) for security *s* at time *t*.

Hence under APT: $R_{s,t} = R_{f,t} + \sum_{k=1}^{K-1} \hat{\beta}_{s,k,t} F_{k,t} + \tilde{\varepsilon}_{s,t}$ … (2.1.2)

where $\tilde{\varepsilon}_{s,t}$ represents the error term with a non-zero average value. *K* is the set of all applicable factors (the risk free rate is an element of that set) and $F_{k,t}$ denotes the observed return from factor *k* at time *t*. Rewriting equation 2.1.2 to ensure that the error term has a zero mean and setting the risk free rate as a factor, gives: $R_{s,t} = \hat{\alpha}_{s,t} + \sum_{k=1}^{K} \hat{\beta}_{s,k,t} F_{k,t} + \varepsilon_{s,t}$ … (2.1.3)

Sharpe (2002) discusses the use of factor models to provide "robust predictions" in risk estimation procedures. We can formally assess a factor model for managers as follows.

Each manager holds a set of securities. Let $\dot{w}_{i,t}^{s}$ denote manager *i*'s holding in security *s* at time *t.* Let *S* be the set of all securities.

Then $\sum_{s=1}^{S} \dot{w}_{i,t}^{s} = 1$

For some mandates, UniSuper allows shorting, as such the usual constraint $\dot{w}_{i,t}^{s} \geq 0 \forall s, s = 1..S$ does not apply in the framework that follows. We can express each manager's expected return as:

$$\mu_{i,t}^{a_i} = \sum_{s=1}^{S} \dot{w}_{i,t}^{s}.\hat{\alpha}_{s,t} + \sum_{s=1}^{S}\sum_{k=1}^{K} \dot{w}_{i,t}^{s}.\hat{\beta}_{s,k,t} F_{k,t} + \sum_{s=1}^{S} \dot{w}_{i,t}^{s}.\varepsilon_{s,t} \quad \text{...(2.1.4)}$$

Where $\mu_{i,t}^{a_i}$ represents manager *i*'s observed returns at time *t*.

Equation 2.4 can be expressed more simply as:

$$\mu_{i,t}^{a_i} = \hat{\alpha}_{i,t}^{a_i} + \sum_{k=1}^{K} \hat{\beta}_{i,k,t} F_{k,t} + \varepsilon_{i,t} \ldots (2.1.5)$$

Where $\hat{\alpha}_{i,t}^{a_i}$ is the estimated weighted *ex-post* average of each stock's idiosyncratic risk (weighted across all securities held by the manager), $\hat{\beta}_{i,k,t}$ is the manager's weighted average exposure to factor *k* at time *t* and $\varepsilon_{i,t}$ is the manager's weighted error term, with a zero mean.

Hence the manager's *ex-post* return consists of an alpha component ($\hat{\alpha}_{i,t}^{a_i}$), a beta component ($\sum_{k=1}^{K} \hat{\beta}_{i,k,t} F_{k,t}$) and an error term ($\varepsilon_{i,t}$). The beta component is estimated by solving for the set of $\left\{\hat{\beta}_{i,k,t}\right\}_{k=1}^{K}$ for each manager, using multiple regression. The formulas presented below were discussed in more detail in Straumann & Garidi (2007).

Equation 2.1.5 can be written in vector-matrix notation: viz. $\overline{\mu} = \overline{X}.\overline{\beta} + \overline{\varepsilon} \ldots (2.1.6)$

$$\text{Or} \begin{bmatrix} \mu_{i,t}^{a_i} \\ \mu_{i,t-1}^{a_i} \\ \ldots \\ \mu_{i,t-r-1}^{a_i} \end{bmatrix} = \begin{bmatrix} 1 & F_{1,t} & F_{2,t} & \ldots & F_{k^*,t} \\ 1 & F_{1,t-1} & F_{2,t-1} & \ldots & F_{k^*,t-1} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & F_{1,t-r-1} & F_{2,t-r-1} & \ldots & F_{k^*,t-r-1} \end{bmatrix} \begin{bmatrix} \hat{\alpha}_{i,t}^{a_i} \\ \hat{\beta}_{i,1,t}^{a_i} \\ \ldots \\ \hat{\beta}_{i,k^*,t}^{a_i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t}^{a_i} \\ \varepsilon_{i,t-1}^{a_i} \\ \ldots \\ \varepsilon_{i,t-r-1}^{a_i} \end{bmatrix}$$

Where $r>k^*$ and *r* denotes the number of months data (both factor and manager data) that is analysed. *r* must be greater than k^* (the number of factors analysed). Here $F_{k,t}$ denotes the *observed* returns from factor *k* at time *t*.

Using multiple regression to solve equation (2.1.6), gives:

$$\hat{\overline{\beta}} = \left(\overline{X}'\overline{X}\right)^{-1}\overline{X}'\overline{\mu} \;\ldots (2.1.7)$$

Equation 2.7 provides an unbiased estimate of the beta factors (noting that the first element in the $\hat{\beta}$ vector represents the *ex-post* alpha estimate for the manager).

A1.2.2. Determining the goodness of fit

The goodness of fit of the regression model given by equation 2.1.7, can be assessed using standard statistical techniques, e.g. by considering the vector of errors arising from the regression at time *t,* which is given by the vector:

$$\hat{\varepsilon} = \overline{\mu} - \overline{X}\hat{\overline{\beta}} \;\ldots (2.2.1)$$

The distribution of errors is carefully examined to ensure that the vector is approximately normally distributed and lacks autocorrelation. TURBOS utilises three common statistical techniques to determine the goodness of fit:

1. A table of statistical values is derived (standard statistical tests are used such as the variance, skewness, kurtosis and autocorrelation of residuals, the model's F-statistic along with R^2 and adjusted R^2);
2. A histogram of residuals relative to a normal distribution is charted; and
3. A normal probability plot is derived.

Once a reasonable fit has been obtained, it is possible to explore the statistical significance of each of the beta factors that were found (as well as the ex-post alpha estimated from the manager) in order to determine the statistical significance of these factors. The formulae that follow for the remainder of this section are presented for completeness and are well-known standard statistics of multiple regression modelling.

The variance of the multiple regression model is given by:

$$\hat{\sigma}_{i,t}^2 = \frac{\hat{\varepsilon}'\hat{\varepsilon}}{r-k^*-1} \quad \ldots (2.2.10)$$

We can also derive the statistical significance and 100(1 − α)% confidence interval of each element within the $\hat{\hat{\beta}}$ vector using *t*-statistics. In particular the 100(1 − α)% confidence interval for the ex-post alpha and each beta factor is computed as follows:

$$\alpha_{i,t}^a = \hat{\alpha}_{i,t}^a \pm t_{\frac{\alpha}{2},r-k^*-1}\hat{\sigma}_{i,t}\sqrt{diag(\overline{X}'\overline{X})_1^{-1}} \quad \ldots (2.2.11a) \text{ and}$$

$$\beta_{i,j,t}^a = \hat{\beta}_{i,j,t}^a \pm t_{\frac{\alpha}{2},r-k^*-1}\hat{\sigma}_{i,t}\sqrt{diag(\overline{X}'\overline{X})_j^{-1}} \text{ for } j = 1..k^* \quad \ldots (2.2.11b)$$

Where $diag(\overline{X}'\overline{X})_1^{-1}$ denotes the first element in the diagonal of the inverse matrix $\overline{X}'\overline{X}$, and $diag(\overline{X}'\overline{X})_j^{-1}$ denotes the j^{th} element in the diagonal of the inverse matrix, for all j, $j=1..K$. Further $t_{\frac{\alpha}{2},r-k^*-1}$ obtained from the Student's *t*-distribution with $r-k^*-1$ degrees of freedom.

Finally, one can assess the statistical significance of each beta factor by considering the t-statistic from each beta factor (given as $\hat{\sigma}_{i,t}\sqrt{diag(\overline{X}'\overline{X})_j^{-1}}$) and then deriving its *p-score* (taken from the inverse cumulative student's *t*-distribution with $r-k^*-1$ degrees of freedom). The observed significance level (or *p-score*) is the smallest fixed level (usually 5%) at which the fitted beta factor is deemed to be statistically significant.

The standard regression approach outlined in section A1.2 is based on the assumption that the factors (F_k) are uncorrelated and have equal and constant uncertainty (i.e. are homoscedastic). In section A1.2.3 we derive a method to handle factors with unstable variances, and in sections 4.2 and A1.2.4 we tackle the more complicated problem of correlation between factors, and the management of co-integration.

A1.2.3. Weighted Least Squares (WLS) estimation

WLS regression compensates for violation of the homoscedasticity assumption by weighting factors (i.e. F_k) differentially. Under WLS, factors which contribute large variances on the regressed manager's returns count less in estimating the $\hat{\beta}_{i,k,t}$ coefficients. The result is that the estimated coefficients are usually very close to what they would be in equation 2.1.7, but under WLS regression their standard errors are smaller. Specifically, the weighted sum of squared residuals is minimised if each factor's weight is equal to the reciprocal of the variance of the factor $\hat{\sigma}_{i,t}^2$. If $\overline{\Phi}$ denotes a square matrix of error estimates for each factor, i.e.

$$\overline{\Phi} = \text{diag}(\hat{\sigma}_{1,t}^2, \hat{\sigma}_{2,t}^2, \ldots, \hat{\sigma}_{k^*,t}^2) \quad \ldots (2.3.1)$$

Then equation 2.1.7 can be rewritten as:

$$\hat{\hat{\beta}} = \left(\overline{X}'\hat{\Phi}^{-1}\overline{X}\right)^{-1}\overline{X}'\hat{\Phi}^{-1}\overline{\mu} \;\ldots\; (2.3.2)$$

And the variance of the estimates amounts to $\left(\overline{X}'\hat{\Phi}^{-1}\overline{X}\right)^{-1}$ …(2.3.3)

Hence, to generate a WLS estimate of the factor exposures, we run the standard multiple regression model provided in section A1.2. To overcome the concerns associated with the lack of homoscedasticity, we re-run the regression formulation, but using a weighted least squares approach outlined in equation 2.3.2. The formulas presented in this section are standard statistical techniques.

A1.2.4. Adjusting the regression solution to overcome collinearity

Collinearity is a statistical concern that arises when two or more factors in a multiple regression model are highly correlated. In this situation, the beta estimates of each manager's returns to the factors (given in equation 2.2.11b above) and the significance tests for the factors (given by their *p-scores*), are underestimated. Further beta estimates change erratically in response to small changes in the data, and the estimates of the *ex-post* alpha become unreliable. Unfortunately, within the finance environment many factors are highly correlated (as an example, monthly returns from the ASX300 Accumulation index was 98% correlated to the ASX listed property index between 2001 and 2007).

To overcome this concern, two alternative approaches to regression analysis were explored and rejected, before we derived an algorithm that is stable and computationally efficient. The first attempt was to utilise principal component analysis (PCA). PCA is a technique used to reduce multidimensional datasets to lower dimensions for analysis, and has the benefit that the new set of factors are orthogonal (hence overcoming the dangers of collinearity). Unfortunately, the generated factors lacked intuitive interpretation and were unstable. The next approach that we explored involved building up the regression model by adding factors until the model became unstable. Hence one would first run the regression with each individual factor and then select the factor with the lowest *p-value* (i.e. the most significant factor) and systematically add factors to the model. Unfortunately this approach frequently converged to a single factor, with low overall model significance (viz. The adjusted R^2 values and F-tests weren't ideal).

We finally derived the following algorithm to overcome collinearity:

1. Generate a multiple regression analysis with all the factors applicable to the asset class;
2. Save the adjusted R^2 of the regression analysis;
3. Find the factor with the highest p-value (i.e. the factor that is least statistically significant);
4. Remove this factor from the set of analysed regression factors;
5. Re-run the regression with all the remaining factors; and
6. Continue the loop *(K-1)* times, until only a single factor remains.

Once all *(K-1)* runs were complete, the TURBOS program scans all the available adjusted R^2 variables to find the highest value. The set of factors that are associated with the optimal adjusted R^2 were then re-run, this time removing any factor whose *p-value* was below the user-defined statistically significant value (usually set at 5%). The end result is that the TURBOS program is able to converge on a set of k^* significant factors that (although correlated) generate stable and intuitive factor exposures for the manager.

A1.3. Estimating the marginal contribution to risk from each manager

Let $\overline{V}$ denote the covariance matrix for the N managers (at time t).

Hence $\overline{V} = \begin{bmatrix} \hat{\sigma}_{11,t} & \dots & \hat{\sigma}_{1N,t} \\ \dots & \dots & \dots \\ \hat{\sigma}_{1N,t} & \dots & \hat{\sigma}_{NN,t} \end{bmatrix}$

Where $\hat{\sigma}_{ij,t}$ denotes the estimated covariance between manager i and manager j at time t.

If $r_{i,t}^{a_i}$ represents manager i's total observed return in month t, *(t=l,l-1,..,l-m+1)* and l is the latest observed time period, then the covariance between manager i and j at time t, is estimated by: $\hat{\sigma}_{ij,t} = \frac{1}{r-1}\sum_{t=1}^{r}\left\{\left(\mu_{i,l-t+1}^{a_i} - \overline{\mu}_{i,l-t+1}^{a_i}\right)\left(\mu_{j,l-t+1}^{a_i} - \overline{\mu}_{j,l-t+1}^{a_i}\right)\right\}$

Where $\overline{\mu}_{i,t-l+1}^{a_i}$ represents the crude average monthly return, hence $\overline{\mu}_{i,t-l+1}^{a_i} = \frac{1}{r}\sum_{l=1}^{r}\mu_{i,t-l+1}^{a_i}$

Note that $\hat{\sigma}_{ij,t}$ assumes that a manager's return series is free of auto-correlation. If $\hat{\rho}_{i,t}^{a_i}$ represents the estimated auto-correlation factor for manager i at time t, then $\hat{\rho}_{i,t}^{a_i}$ can be roughly estimated, using a one-month lag, as follows:

$$\hat{\rho}_{i,t}^{a_i} = \frac{\sum_{t=1}^{r-1}\left\{\left(\mu_{i,l-t+1}^{a_i} - \overline{\mu}_{i,l-t+1}^{a_i}\right)\left(\mu_{i,l-t+2}^{a_i} - \vec{\mu}_{i,l-t+1}^{a_i}\right)\right\}}{\sum_{t=1}^{r}\left(\mu_{i,l-t+1}^{a_i} - \overline{\mu}_{i,l-t+1}^{a_i}\right)^2}$$

And $\hat{\sigma}_{ij,t}$ must be adjusted for autocorrelation by dividing the observed covariance between manager i and manager j by the factor $\left(1-\hat{\rho}_{i,t}^{a_i}\right)\left(1-\hat{\rho}_{j,t}^{a_i}\right)$. Note that if no evidence of auto-correlation exists, then $\hat{\rho}_{i,t}^{a_i}$ =0 and no adjustment is required to the standard covariance formula. The approach generates a practical and conservative adjustment to the covariance matrix, but may well overstate the covariance of returns for listed equity managers who utilise momentum strategies.

Hence covariance of returns is estimated as:

$$\hat{\sigma}_{ij,t} = \frac{\sum_{t=1}^{r}\left\{\left(\mu_{i,l-t+1}^{a_i} - \overline{\mu}_{i,l-t+1}^{a_i}\right)\left(\mu_{j,l-t+1}^{a_i} - \overline{\mu}_{j,l-t+1}^{a_i}\right)\right\}}{(r-1)\left(1-\hat{\rho}_{i,t}^{a_i}\right)\left(1-\hat{\rho}_{j,t}^{a_j}\right)} \quad \dots(3.1)$$

Further, each Option's variance at time t ($\hat{\sigma}_{o,t}^2$) can be estimated from the ex-post data:

$$\hat{\sigma}_{o,t}^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{i,t}^{o,a_i} w_{j,t}^{o,a_j}\hat{\sigma}_{ij,t} = \overline{w}'\overline{V}\overline{w} \quad \dots(3.2)$$

Where $\overline{w}$ is defined in equation (1.1) above.

The marginal contribution to the total fund variance, for manager *i* is derived by taking the partial derivative of the total option variance ($\overline{w}'\overline{V}\overline{w}$) with respect to the weighting for manager *i*:

$$\frac{\delta\sigma_{o,t}^2}{\delta w_{i,t}^{o,a}} = \frac{\delta \overline{w}'\overline{V}\overline{w}}{\delta w_{i,t}^{o,a}} = \frac{\delta \sum_{i=1}^{N} \left(w_{i,t}^{o,a_i}\right)^2 \hat{\sigma}_{ii,t}^2}{\delta w_{i,t}^{o,a}} + \frac{\delta \sum_{i=1}^{N} \sum_{j=1,i<>j}^{N} w_{i,t}^{o,a_i} w_{j,t}^{o,a_j} \hat{\sigma}_{ij,t}}{\delta w_{i,t}^{o,a}} = 2w_{i,t}^{o,a_j}\hat{\sigma}_{ii,t}^2 + \sum_{j=1,i<>j}^{N} w_{j,t}^{o,a_j}\hat{\sigma}_{ij,t}$$

$$\Rightarrow \frac{\delta \overline{w}'\overline{V}\overline{w}}{\delta w_{i,t}^{a}} = 2\sum_{j=1}^{N} w_{j,t}^{o,a_j}\hat{\sigma}_{ij,t} \quad \ldots(3.3)$$

Equation 3.3 allows us to derive the marginal contribution to the total option risk (taken as the standard deviation of returns for Option *o*, or $\hat{\sigma}_{o,t}$)

$$\frac{\delta\hat{\sigma}_{o,t}}{\delta w_{i,t}^{o,a}} = \frac{1}{2\hat{\sigma}_{o,t}} \frac{\delta\sigma_{o,t}^2}{\delta w_{i,t}^{o,a}} = \frac{1}{2\sigma_{o,t}} \frac{\delta \overline{w}'\overline{V}\overline{w}}{\delta w_{i,t}^{o,a}} = \frac{2\sigma_{io,t}}{2\sigma_{o,t}} = \frac{\sigma_{io,t}}{\sigma_{o,t}} \quad \ldots(3.5)$$

Hence the marginal contribution to the total option risk from manager *i* at time *t* is the ratio of the covariance of returns between manager *i* and the option, divided by the standard deviation of returns of the option.

Each manager's proportional contribution to risk is simply their marginal contribution multiplied by their weight in the portfolio. This is the approach outlined by Mina (2007).

Hence proportional contribution to risk for Manager *i* = $w_{i,t}^{o,a_i}\dfrac{\sigma_{io,t}}{\sigma_{o,t}}$ ...(3.6)

As a check, the sum of the proportional contribution to risk per manager, equals the option's standard deviation of return: $\sum_{i=1}^{N} w_{i,t}^{o,a_i}\dfrac{\sigma_{io,t}}{\sigma_{o,t}} = \dfrac{1}{\sigma_{o,t}}\sum_{i=1}^{N} w_{i,t}^{o,a_i}\sum_{j=1}^{N} w_{j,t}^{o,a_j}\hat{\sigma}_{ij,t} = \dfrac{\sigma_{o,t}^2}{\sigma_{o,t}} = \sigma_{o,t}$ …(3.7)

Equations 3.6 and 3.7 were derived by Scherer (2000) and Ilricht (2004).

A1.4. Determining Each Manager's Ex-Ante Alpha

We start by considering utility theory, which can be utilised to assess equilibrium expected returns, and then re-written to find the optimal portfolio given a covariance matrix and a vector of expected returns.

Such an approach introduces the need for a Bayesian framework to determine ex-ante returns. After reviewing commonly used methods to find ex-ante return estimates, we selected the Black-Litterman Model (BLM), which was adapted to consider alpha, rather than total return expectations. The approach used is similar to that considered by Winkelman (Litterman, 2003b) except that we did not make the simplifying assumption that alpha is independent between managers. The assumption that alpha is independent between managers occurs frequently within risk budgeting papers. Whilst alpha and beta are independent (by design), alpha between similar manager styles is frequently correlated (e.g. when a single quantitative managers outperforms, frequently many quantitative managers also outperform, partly as similar methodologies are selected by managers). As such, we felt it necessary to consider the possibility that managers' alpha could be correlated.

A Markowitz efficient portfolio is one that offers the greatest expected return for a given level of risk. To find the set of such portfolios, a computationally efficient approach is to maximise the Fund's total return per unit of risk. To do so we make use of utility theory and calculus. Utility theory (outlined in a variety of papers e.g. Mina (2007)), generates a variety of key findings, depending on the shape of the investor's utility function, but one key conclusion, states that the investment objective is to maximise expected return per unit of risk.

$$\text{Maximise: } \overline{w}'\mu - \frac{\lambda}{2}\overline{w}'\overline{V}\overline{w} \ \ldots\ (4.1)$$

Where λ denotes the investor's risk aversion parameter.

Taking the derivative of 4.1 with respect to the weight for manager *i* and setting this equation to zero, along with making use of equation 3.3 gives:

$$\frac{\delta\,\overline{w}'\mu}{\delta\, w_{i,t}^{a}} - \frac{\lambda}{2}\frac{\delta\,\overline{w}'\overline{V}\overline{w}}{\delta\, w_{i,t}^{a}} = \mu_{i,t}^{a} - \frac{\lambda}{2}2\sum_{j=1}^{N} w_{j,t}^{a_j}\hat{\sigma}_{ij,t} = \mu_{i,t}^{a} - \lambda\sum_{j=1}^{N} w_{j,t}^{a_j}\hat{\sigma}_{ij,t} = 0 \ \ldots(4.2)$$

Hence the local maximum of 4.1 for manager *i* is derived when $\mu_{i,t}^{a_i} = \lambda\sum_{j=1}^{N} w_{j,t}^{a_j}\hat{\sigma}_{ij,t}$ …(4.3)

Note that $\lambda\sum_{j=1}^{N} w_{j,t}^{a_j}\hat{\sigma}_{ij,t}$ represents the i^{th} element of the vector $\lambda\overline{V}\,\overline{w}$. If $\overline{w}^{*}$ denotes the vector of optimal manager weights, then the local maximum of 4.1 is derived when:

$$\overline{\mu} = \lambda\overline{V}\overline{w}^{*} \ \ldots(4.4)$$

Pre-multiplying (4.4) by the inverse of the covariance matrix gives:

$$\overline{w}^{*} = \frac{1}{\lambda}\overline{V}^{-1}\overline{\mu} \ \ldots(4.5).$$

A1.4.1. Reverse optimisation

One can use equation 4.4 to derive the expected return for an Option *o*, given the estimated risk aversion parameter for the Option ($\dot{\lambda}^{o}$), the estimated variance-covariance matrix ($\hat{\overline{V}}$), and the current manager weights within the option portfolio, viz. $\hat{\overline{\mu}} = \dot{\lambda}^{o}\hat{\overline{V}}\overline{w}$ …(4.6).

Equations 4.5 and 4.6 have been presented in a variety of papers, e.g. Mina (2007).

As the risk aversion parameter for each Accumulation Option cannot be accurately ascertained, equation 4.6 is only of benefit to determine the *relative differences* between expected returns for managers. If one is able to derive the expected return for a single manager with a high degree of confidence, using prior knowledge, then the set of total expected returns can be determined. An obvious candidate for the selection of the base manager is a cash manager where returns are generally stable and predictable.

The above paragraph introduces a Bayesian consideration into the determination of ex-ante return forecasts. In addition, equation 4.6 ignores the uncertainty of expected returns in equilibrium.

To allow for the investor's views as well as the need to deal with the uncertainty of equilibrium expected returns have been considered using the Black-Litterman Model (Litterman, 2003).

A1.4.2. The Black-Litterman Model (BLM)

The BLM deals with the uncertainty of expected returns by adopting a Bayesian approach, where the investor's views is subject to error and can be modified by the market's initial equilibrium return expectation, to derive a blended return expectation.

BL define $\overline{w}_{eq}$ as the equilibrium portfolio of all assets in the market (weighted by market capitalisation). If there are N^* assets in the universe of all assets, then $\overline{w}_{eq}$ is a 1 x N^* vector.
The covariance matrix for all assets is defined as $\overline{\Sigma}$ (an N^* x N^* matrix).

Under the BL Model, the implied equilibrium of returns (in excess of the risk free rate) is normally distributed with an expected return vector of Π:

$$E[\overline{\Pi}] = \delta \overline{\Sigma}\, \overline{w}_{eq} \ldots (4.2.1)$$

Equation 4.2.1 is derived in an analogous manner to equation 4.1 (i.e. by determining the maximum return per unit of risk). In equation 4.2.1, δ represents the world-wide risk aversion parameter, and is usually set to the risk-regression slope of all asset classes. The variance of the equilibrium return is given by $\overline{\Sigma}$ and in the BL model is scaled down by a factor τ which measures the uncertainty of the *priori*.

Hence the market's prior equilibrium distribution of returns ($\overline{\Pi}$) is distributed Normally i.e.

$$\overline{\Pi} \sim N(\ \delta \overline{\Sigma}\, \overline{w}_{eq}\ ,\ \tau\overline{\Sigma}\ )\ \ \ldots (4.2.2)$$

Having determined the prior distribution of equilibrium returns, the BL model then allows for the investor's views. If the investor has K views (clearly, $K \leq N^*$), which are independent of each other and of the market's prior equilibrium, then the BL model determines the investor's expected overall returns given their views and the confidence the investor has in their views. Let $\overline{Q}$ denote the matrix of investor's views (a K x N^* matrix). Then $\overline{Q}$ can be expressed as the product of $\overline{\mathrm{P}}$ (which is a K x N^* matrix) and the prior equilibrium distribution of returns, $\overline{\Pi}$. Further if $\overline{\Omega}$ denotes the investor's confidence in their views, then $\overline{Q}$ is normally distributed with mean $\overline{\mathrm{P}\Pi}$, and variance $\overline{\Omega}$.

$$\text{Specifically: } \overline{Q} \sim N(\ \overline{\mathrm{P}\Pi}\ ,\ \overline{\Omega}\ )\ \ \ldots (4.2.3)$$

We wish to find the Combined return distribution of returns, given the investor's views. The result is presented in equation 4.2.6 and was derived by Black and Litterman (see for example Litterman (2003a)). The following page describes one method of deriving the BLM from first principles, based on work by Jiang *et al* (2005).

If we let $\overline{Y} = \begin{bmatrix} \overline{\Pi} \\ \overline{Q} \end{bmatrix}$, $\overline{X} = \begin{bmatrix} \overline{I} \\ \overline{\mathrm{P}} \end{bmatrix}$ and $\overline{\varepsilon} = \begin{bmatrix} \tau\overline{\Sigma} \\ \overline{\Omega} \end{bmatrix}$ then the regression solution to $\overline{Y} = \overline{X}\,\overline{\Pi} + \overline{\varepsilon}$ (3) which minimises the error term $\overline{\varepsilon}$ would provide the optimal solution to equations 4.2.2 and 4.2.3 and would generate a weighted average of the equilibrium returns and the investor's views (or, using Bayesian terminology, the posterior distribution of $\overline{\Pi}$ given $\overline{Q}$). The greater the investor's confidence in $\overline{Q}$ (i.e. the lower the values in $\overline{\Omega}$), the higher the weight of the combined views to the investor's view.

We also know that $\overline{\varepsilon} \sim N\left(0, \begin{bmatrix} \tau\overline{\Sigma} & 0 \\ 0 & \overline{\Omega} \end{bmatrix}\right)$. If we let $\Theta = \begin{bmatrix} \tau\overline{\Sigma} & 0 \\ 0 & \overline{\Omega} \end{bmatrix}$ then, using weighted least squares, the solution to: $\overline{\mu} = \overline{X}.\overline{\beta} + \overline{\varepsilon}$ is provided by $\hat{\overline{\beta}}$ which has an unbiased expected value of $\left(\overline{X}'\hat{\Phi}^{-1}\overline{X}\right)^{-1}\overline{X}'\hat{\Phi}^{-1}\overline{\mu}$ and an estimated variance of $\left(\overline{X}'\hat{\Phi}^{-1}\overline{X}\right)^{-1}$ (Refer equation 2.2.2 and 2.2.3, and note that $\overline{\Phi}$ denotes a square matrix of error estimates for each factor).

By substitution, we therefore have that the unbiased estimate of the combined view is: $\left(\overline{X}'\overline{\Theta}^{-1}\overline{X}\right)^{-1}\overline{X}'\overline{\Theta}^{-1}\overline{Y}$, while the variance of the combined view is $\left(\overline{X}'\overline{\Theta}^{-1}\overline{X}\right)^{-1}$.

Hence, the unbiased estimate of the combined view

$$= \left(\left[\overline{I}\ \overline{\mathrm{P}}'\right] \begin{bmatrix} \tau\overline{\Sigma} & 0 \\ 0 & \overline{\Omega} \end{bmatrix}^{-1} \begin{bmatrix} \overline{I} \\ \overline{\mathrm{P}} \end{bmatrix} \right)^{-1} \left[I\overline{\mathrm{P}}'\right] \begin{bmatrix} \tau\overline{\Sigma} & 0 \\ 0 & \overline{\Omega} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\Pi} \\ \overline{Q} \end{bmatrix}$$

And the variance of the combined view $= \left(\left[\overline{I}\ \overline{\mathrm{P}}'\right] \begin{bmatrix} \tau\overline{\Sigma} & 0 \\ 0 & \overline{\Omega} \end{bmatrix}^{-1} \begin{bmatrix} \overline{I} \\ \overline{\mathrm{P}} \end{bmatrix} \right)^{-1}$

Starting with the solution to the variance, gives:

$$\text{Variance} = \left(\left[\overline{I}\ \overline{\mathrm{P}}'\right] \begin{bmatrix} (\tau\overline{\Sigma})^{-1} & 0 \\ 0 & \overline{\Omega}^{-1} \end{bmatrix} \begin{bmatrix} \overline{I} \\ \overline{\mathrm{P}} \end{bmatrix} \right)^{-1} = \left(\left[(\tau\overline{\Sigma})^{-1}\ \overline{\mathrm{P}}\,\overline{\Omega}^{-1}\right] \begin{bmatrix} \overline{I} \\ \overline{\mathrm{P}} \end{bmatrix} \right)^{-1} = \left[(\tau\overline{\Sigma})^{-1} + \overline{\mathrm{P}}\,\overline{\Omega}^{-1}\overline{\mathrm{P}}\right]^{-1} \ldots(4.2.4)$$

We can also solve for the unbiased estimate of the combined view:

$$= \left(\left[\overline{I}\ \overline{\mathrm{P}}'\right] \begin{bmatrix} \tau\overline{\Sigma} & 0 \\ 0 & \overline{\Omega} \end{bmatrix}^{-1} \begin{bmatrix} \overline{I} \\ \overline{\mathrm{P}} \end{bmatrix} \right)^{-1} \left[I\overline{\mathrm{P}}'\right] \begin{bmatrix} \tau\overline{\Sigma} & 0 \\ 0 & \overline{\Omega} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\Pi} \\ \overline{Q} \end{bmatrix}$$

$$= \left[(\tau\overline{\Sigma})^{-1} + \overline{\mathrm{P}}\,\overline{\Omega}^{-1}\overline{\mathrm{P}}\right]^{-1} \left[I\overline{\mathrm{P}}'\right] \begin{bmatrix} \tau\overline{\Sigma} & 0 \\ 0 & \overline{\Omega} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\Pi} \\ \overline{Q} \end{bmatrix}$$

$$= \left[(\tau\overline{\Sigma})^{-1} + \overline{\mathrm{P}}\,\overline{\Omega}^{-1}\overline{\mathrm{P}}\right]^{-1} \left[(\tau\overline{\Sigma})^{-1}\ \overline{\mathrm{P}}\,\overline{\Omega}^{-1}\right] \begin{bmatrix} \overline{\Pi} \\ \overline{Q} \end{bmatrix}$$

$$= \left[(\tau \overline{\Sigma})^{-1} + \overline{\mathrm{P}}\,\overline{\Omega}^{-1}\overline{\mathrm{P}} \right]^{-1} \left[(\tau \overline{\Sigma})^{-1}\overline{\Pi} + \overline{\mathrm{P}}\,\overline{\Omega}^{-1}\overline{Q} \right] \ldots (4.2.5)$$

Hence, the BLM generates a combined view that is Normally distributed with an expected value given in equation 4.2.5 and a variance given by equation 4.2.4.

The combined view $\sim N\left(\left[(\tau \overline{\Sigma})^{-1} + \overline{\mathrm{P}}\,\overline{\Omega}^{-1}\overline{\mathrm{P}} \right]^{-1} \left[(\tau \overline{\Sigma})^{-1}\overline{\Pi} + \overline{\mathrm{P}}\,\overline{\Omega}^{-1}\overline{Q} \right], \left[(\tau \overline{\Sigma})^{-1} + \overline{\mathrm{P}}\,\overline{\Omega}^{-1}\overline{\mathrm{P}} \right]^{-1} \right) \ldots (4.2.6)$

Equation 4.2.6 has been presented in numerous papers, including Litterman (2003a), Walters (2008), Meucci (2008) etc.

A1.4.3. Adapting the BLM for active risk

Equation 4.2.6 provides a framework for combining equilibrium returns ($\overline{\Pi} = \delta \overline{\Sigma}\, \overline{w}_{eq}$) for N* assets, *K* investor specific views ($\overline{\mathrm{P}}$), the weight the investor places on the equilibrium view $\left(\frac{1}{\tau}\right)$ and the confidence the investor has in their views ($\overline{\Omega}$). The BLM can be extended to consider active risk, as follows.

If

$E[\hat{\alpha}_{i,t}^{a_i}]$	Denotes the ex-ante expected alpha from manager *i*, at time *t*.
$\hat{\overline{\alpha}}$	Denotes the vector of expected alphas.
$\hat{\overline{\Psi}}$	Denotes the covariance matrix of each manager's returns in excess of their factor exposures.
$\overline{Q}_A$	Denotes the investor's view of the expected alpha generated from each manager.
$\overline{\Omega}_A$	Denotes the confidence the investor has in each manager's ability to generate alpha.

Then $\hat{\overline{\alpha}} = \left[\left(\tau \hat{\overline{\Psi}}\right)^{-1} + \overline{\mathrm{P}}_A\,\overline{\Omega}_A^{-1}\overline{\mathrm{P}}_A \right]^{-1} \left[\left(\tau \hat{\overline{\Psi}}\right)^{-1}\overline{\Pi}_A + \overline{\mathrm{P}}_A\,\overline{\Omega}_A^{-1}\overline{Q}_A \right] \ldots (4.2.7)$

Where $\overline{\Pi}_A$ denotes the vector of equilibrium active returns.

When considering total returns (i.e. in equation 4.2.6), $\overline{Q}$ is usually expressed for convenience as the product of $\overline{\mathrm{P}}$ (which is a *K* x *N** matrix) and the prior equilibrium distribution of returns, as the investor often has a view that a certain asset class will outperform another asset class, but has less confidence as to the absolute return generated by both asset classes. However, one is not required to express $\overline{Q}$ as a product of $\overline{\mathrm{P}}$ and $\overline{\Pi}$, and when considering active returns the investor tends to have a view as to the absolute level of alpha generated by each asset or manager. Litterman (2003b) derived equation 4.2.7 and explains that the equation can be further simplified by considering that in equilibrium:

a) Active returns for all assets equal zero (i.e. $\overline{\Pi}_A = \overline{0}$); and
b) The investor's views about expected alpha (given in $\overline{Q}_A$) are formed independently to equilibrium returns ($\overline{\Pi}_A$), hence $\overline{\mathrm{P}}_A$ is an identity matrix.

Hence $\hat{\overline{\alpha}} = \left[\left(\tau \hat{\overline{\Psi}}\right)^{-1} + \overline{\Omega}_A^{-1} \right]^{-1} \left[\overline{\Omega}_A^{-1}\overline{Q}_A \right] \ldots (4.2.8)$

Equation 4.2.8 is presented in Litterman (2003b) and is central to UniSuper's risk-budgeting framework, as it relates expected active returns to UniSuper's views about active returns, UniSuper's confidence in those views and the covariance between historic active returns.

Note that equation 4.2.7 is also of use, if historic alpha estimates replace the equilibrium alpha estimate for each manager. Under this scenario, the investor's views about expected alpha ($\overline{Q}_A$) are still formed independently to historic returns ($\hat{\overline{\Pi}}_A$), and $\overline{\mathrm{P}}_A$ remains an identity matrix. If we let $\overline{\mathrm{A}}$ denote the vector of ex-post alpha estimates, then $\overline{\mathrm{A}} = \begin{bmatrix} \alpha_{1,t}^{a_1} \\ \ldots \\ \alpha_{N,t}^{a_N} \end{bmatrix}$ where $\alpha_{s,t}^{a_i}$ is derived from each manager's regression analysis provided in equation 2.4, then we can recast equation 4.2.7 as follows:

$$\hat{\alpha} = \left[\left(\tau\hat{\Psi}\right)^{-1} + \overline{\Omega}_A^{\ -1} \right]^{-1} \left[\left(\tau\hat{\Psi}\right)^{-1} \overline{\mathrm{A}} + \ \overline{\Omega}_A^{\ -1} \overline{Q}_A \right] \ldots(4.2.9)$$

The results from equation 4.2.9 can be contrasted to that of 4.2.8 as it relates expected active returns to the investor's views about each manager's active returns, their confidence in those views, the covariance between historic active returns, the actual observed ex-post alpha and the confidence the investor places on those historic ex-post estimates, given by $\left(\frac{1}{\tau}\right)$.

Effectively, equation 4.2.9 generates en *ex-ante* alpha that is a weighted average of historic ex-post alpha estimates combined with the investor's views. Whilst UniSuper uses equation 4.2.9 within TURBOs, equation 4.2.8 is an appropriate alternative.

A1.5. Return Attribution

By combining and weighting the Fund's exposure to all *N* managers we can derive the ex-ante expected return for Option (*o*) and attribute this return between a variety of sources. In section A1.2 we attributed the each manager's total return ($\mu_{i,t}^{a_i}$), to an alpha and beta component, viz.

$$\mu_{i,t}^{a_i} = \hat{\alpha}_{i,t}^{a_i} + \sum_{k=1}^{K} \hat{\beta}_{i,k,t} F_{k,t} + \varepsilon_{i,t}$$

Where $\hat{\beta}_{i,k,t}$ is the manager's derived exposure to Factor *k* at time *t* and $\varepsilon_{i,t}$ is the manager's weighted error term, with a zero mean.

Return attribution has been discussed by a variety of authors (including Mina (2007) and Litterman (2003b)). We extend the analysis to consider how each of the manager's regressed factors map onto the Fund's SAA benchmarks.

Within the set of available beta factors $\{F_k\}_{k=1}^{K}$ there is a sub-set of *M* factors that equate to the Fund's benchmark for each asset class (as an example, for the Australian Equity asset class, the current benchmark is the ASX 300 Accumulation index – which in turn is a potential beta factor). Let $\{BM^*{}_m\}_{m=1}^{M}$ denote the set of strategic benchmark factors, for each of the Fund's *M* asset classes.

Then we can determine the manner in which each beta factor maps onto the Fund's strategic benchmark factors as follows:

$$F_{k,t} = RP_k + \sum_{m=1}^{M} \hat{\gamma}_{m,k}\ BM^*_{m,t} + \nu_{k,t} \ldots(5.1)$$

where RP_k denotes the risk premium available from Factor *k* which cannot be attributed to one of the Fund's benchmarks, $\hat{\gamma}_{m,k}$ denotes the estimated sensitivity between factor *k* and the m^{th} asset class benchmark. $\nu_{k,t}$ denotes the residuals of the regression equation, with a zero mean. The risk premia, and the $\hat{\gamma}_{m,k}$ factors are solved in the same manner that was used to derive the factors from equation 2.1.5 (viz. weighted least squares multiple regression, with the same optimisation algorithm to manage collinearity).

From equation 2.1.5, we can express each manager's returns as a function of $\alpha^{a_i}_{i,t}$, along with a set of factor exposures $\sum_{k=1}^{K} \hat{\beta}_{i,k,t}\ F_{k,t}$. Equation 5.1 allows us to assess the link between each factor exposure to that of the Fund's SAA benchmarks.

By combining equations 2.1.5, 4.2.9 and 5.1, we can derive the expected return for each option expressed as a function of:

e) The weighted average of each manager's expected ex-ante alpha;
f) The weighted average exposure to the Fund's SAA benchmark;
g) Extraneous beta risk premia from factors that differ to the Fund's SAA benchmark; and
h) An error term.

Let R^o_t denote the historic (ex-post) return from Option *o* at time *t*.

Then, $R^o_t = \sum_{i=1}^{N} w^{o,a_i}_{i,t}\ \mu^{a_i}_{i,t} \ldots(5.2)$

Substituting equation 2.1.5 into 5.1 gives:

$$R^o_t = \sum_{i=1}^{N} w^{o,a_i}_{i,t}\hat{\alpha}^{a_i}_{i,t} + \sum_{i=1}^{N} w^{o,a_i}_{i,t}\sum_{k=1}^{K}\hat{\beta}_{i,k,t}F_{k,t} + \sum_{i=1}^{N} w^{o,a_i}_{i,t}\varepsilon_{i,t} \ldots(5.3)$$

Substituting equation 5.1 into equation 5.3 gives:

$$R^o_t = \sum_{i=1}^{N} w^{o,a_i}_{i,t}\hat{\alpha}^{a_i}_{i,t} + \sum_{i=1}^{N} w^{o,a_i}_{i,t}\sum_{k=1}^{K}\hat{\beta}_{i,k,t}\left\{RP_k + \sum_{m=1}^{M}\hat{\gamma}_{m,k}\ BM^*_{m,t} + \nu_{k,t}\right\} + \sum_{i=1}^{N} w^{o,a_i}_{i,t}\varepsilon_{i,t} \ldots(5.4)$$

$$R^o_t = \left\{\sum_{i=1}^{N} w^{o,a_i}_{i,t}\hat{\alpha}^{a_i}_{i,t}\right\} + \left\{\left[\sum_{i=1}^{N}\sum_{k=1}^{K}\sum_{m=1}^{M} w^{o,a_i}_{i,t}\hat{\gamma}_{m,k}\hat{\beta}_{i,k,t}BM^*_{m,t}\right] + \left[\sum_{i=1}^{N}\sum_{k=1}^{K} w^{o,a_i}_{i,t}\hat{\beta}_{i,k,t}RP_k\right]\right\} + \left\{\sum_{i=1}^{N}\sum_{k=1}^{K} w^{o,a_i}_{i,t}\hat{\beta}_{i,k,t}\nu_{k,t} + \sum_{i=1}^{N} w^{o,a_i}_{i,t}\varepsilon_{i,t}\right\} \ldots(5.5)$$

The above equation is central to the Fund's factor analysis. In particular:

- $\left\{\sum_{i=1}^{N} w^{o,a_i}_{i,t}\hat{\alpha}^{a_i}_{i,t}\right\}$ denotes the ex-post alpha expected from the fund's managers for Option *o*.
- The term $\left[\sum_{i=1}^{N}\sum_{k=1}^{K}\sum_{m=1}^{M} w^{o,a_i}_{i,t}\hat{\gamma}_{m,k}\hat{\beta}_{i,k,t}BM^*_{m,t}\right]$ denotes the weighted average exposure to the Option's SAA benchmark;

- While $\left[\sum_{i=1}^{N}\sum_{k=1}^{K} w_{i,t}^{o,a_i}\hat{\beta}_{i,k,t}RP_k\right]$ denotes the Option's exposure to other known beta sources (such as credit risk, value and small cap biases, sector bets etc);
- The final term $\left\{\sum_{i=1}^{N}\sum_{k=1}^{K} w_{i,t}^{o,a_i}\hat{\beta}_{i,k,t}\nu_{k,t} + \sum_{i=1}^{N} w_{i,t}^{o,a_i}\varepsilon_{i,t}\right\}$, represents the error or residuals of the various regression estimates (and has a zero mean). The distribution is critically examined to ensure that it is sufficiently normally distributed. Evidence of extreme kurtosis or skewness, could be cause for concern.

Recall that in section 1 we defined W_m^o as the time-independent Strategic Asset Allocation (SAA) weight for Option *o* within asset class *m*. Hence if the Fund were to passively match its SAA, then the expected return for Option *o*, would amount to: $\sum_{m=1}^{M} W_m^o E[BM_{m,t}^*]$ …(5.6)

Where $E\left[BM_{m,t}^*\right]$ denotes the expected return for the benchmark factor for asset class *m* at time *t*.

Let RA_t^o denote the Fund's total actual risk allocation for Option *o* at time *t*. Hence RA_t^o represents the deviation from the Fund's SAA for Option *o* at time *t*.

Then $RA_t^o = \sum_{i=1}^{N} w_{i,t}^{o,a_i}\mu_{i,t}^{a_i} - \sum_{m=1}^{M} W_m^o BM_{m,t}^*$ …(5.7)

$$=> RA_t^o$$

$$= \left\{\sum_{i=1}^{N} w_{i,t}^{o,a_i}\hat{\alpha}_{i,t}^{a_i}\right\} + \left\{\left[\sum_{m=1}^{M}\sum_{i=1}^{N}\sum_{k=1}^{K} w_{i,t}^{o,a_i}\hat{\beta}_{i,k,t}\hat{\gamma}_{m,k}BM_{m,t}^* - \sum_{m=1}^{M} W_m^o BM_{m,t}^*\right] + \left[\sum_{i=1}^{N}\sum_{k=1}^{K} w_{i,t}^{o,a_i}\hat{\beta}_{i,k,t}RP_k\right]\right\} + \varepsilon$$

Where $\varepsilon = \left\{\sum_{i=1}^{N}\sum_{k=1}^{K} w_{i,t}^{o,a_i}\hat{\beta}_{i,k,t}\nu_{k,t} + \sum_{i=1}^{N} w_{i,t}^{o,a_i}\varepsilon_{i,t}\right\}$ ...(5.8)

Hence RA_t^o = *{Active Management Risk}* + *{Factor Risk}* + *{Error Distribution Analysis}*

As such, each Option is exposed to three forms of tracking error or risk. The first relates to active management risk, the second and most significant risk relates to the extent that the beta Factor exposures for the Option differs to the Option's SAA, whilst the third risk relates to the existence of other extraneous beta factors within the portfolio.

In addition to risk attribution, one can calculate the expected return for each Option. By taking the expectation of equation 5.8, and noting that the expected error term is zero (i.e $E[\varepsilon]=0$).

$$E[R_t^o] = \left\{\sum_{i=1}^{N} w_{i,t}^{o,a_i}E[\hat{\alpha}_{i,t}^{a_i}]\right\} + \left\{\left[\sum_{i=1}^{N}\sum_{k=1}^{K}\sum_{m=1}^{M} w_{i,t}^{o,a_i}\hat{\gamma}_{m,k}\hat{\beta}_{i,k,t}E[BM_{m,t}^*]\right] + \left[\sum_{i=1}^{N}\sum_{k=1}^{K} w_{i,t}^{o,a_i}\hat{\beta}_{i,k,t}E[RP_k]\right]\right\}$$

...(5.9)

Where $E[\hat{\alpha}_{i,t}^{a_i}]$ denotes each manager's expected *ex-ante* alpha (derived in section 5 of this appendix).

TURBOS determines each Option's *ex-ante* alpha value, and then tests whether the estimate exceeds the minimum required hurdle.

For each Option, $E[R_t^o]$ is contrasted to the Option's investment objectives, to ensure that the Fund remains confident that the objectives remain achievable with appropriate levels of confidence.

A1.6. Risk Allocation

If we let $\hat{\bar{\alpha}}$ denote the vector of estimated ex-ante alpha's for each manager, and $\hat{\bar{\Psi}}$ denote the estimated variance-covariance matrix of excess returns, then

$$\hat{\bar{\alpha}} = \begin{bmatrix} \hat{\alpha}_{1,t}^{AEQ} \\ \hat{\alpha}_{2,t}^{AEQ} \\ \dots \\ \hat{\alpha}_{N,t}^{CA} \end{bmatrix} \text{ and } \hat{\bar{\Psi}} = \begin{bmatrix} \hat{\psi}_{11,t} & \dots & \hat{\psi}_{1N,t} \\ \dots & \dots & \dots \\ \hat{\psi}_{1N,t} & \dots & \hat{\psi}_{NN,t} \end{bmatrix} \dots(5.6)$$

Where $\hat{\psi}_{ij,t}$ denotes the estimated covariance between manager *i* and manager *j*'s *excess* returns, or alpha streams. Generally, one expects $\hat{\psi}_{ij,t} = 0$ when *i<>j*. However, importantly, we have not forced or assumed these cross-correlations to be zero and $\sqrt{\hat{\psi}_{ii,t}}$ denotes manager *i*'s tracking error relative to the manager's observed or fitted beta exposures. $\overline{TE}$ denotes the vector of tracking errors for each manager at time *t*. i.e. $\overline{TE}' = \left\lfloor \sqrt{\hat{\psi}_{11,t}}, \sqrt{\hat{\psi}_{22,t}}, \quad \dots \quad, \sqrt{\hat{\psi}_{ii,t}} \right\rfloor$.

As a result, $\hat{\bar{\Psi}}$ can be rewritten as $\overline{TE}'.\overline{C}.\overline{TE}$, where $\overline{C}$ denotes the correlation matrix of excess returns, which need not equal the identity matrix.

The optimal active risk allocation problem can be specified as maximising the Fund's expected active return, subject to a specified risk budget. Risk allocation typically focuses on setting a limit on the Fund' tracking error. Although this is not the approach adopted by TURBOs, it is useful to assess each Option's risk allocation using the simpler statistic of a tracking error.

We start with the traditional solution to risk budgeting namely to maximise expected excess returns subject to the constraint that the Fund's tracking error remains within a specified limit. Algebraically this problem can be specified as: Maximise: $\overline{w}'\hat{\bar{\alpha}}$, subject to the constraint that Option *o*'s active tracking error is less than a specified maximum (i.e. $\sqrt{\overline{w}'\hat{\bar{\Psi}}\overline{w}} \leq TE_{\max}$), where $TE_{\max}$ represents the pre-determined maximum permissible tracking error.

From section 4.1 we know that the maximum of $\overline{w}'\overline{\mu} - \frac{\lambda}{2}\overline{w}'\overline{V}\overline{w}$ occurs, when the optimal portfolio (given by $\overline{w}^*$) equals $\frac{1}{\lambda}\overline{V}^{-1}\overline{\mu}$.

By substitution, the function $\overline{w}'\hat{\bar{\alpha}} - \frac{\lambda}{2}\overline{w}'\hat{\bar{\Psi}}\overline{w}$ is maximised if $\overline{w}^* = \frac{1}{\lambda}\hat{\bar{\Psi}}^{-1}\hat{\bar{\alpha}}$ …(6.1)

We now need to solve for λ such that the constraint is obtained. Hence

λ is such that: $\sqrt{\left(\frac{1}{\lambda}\hat{\bar{\Psi}}^{-1}\hat{\bar{\alpha}}\right)'\hat{\bar{\Psi}}\left(\frac{1}{\lambda}\hat{\bar{\Psi}}^{-1}\hat{\bar{\alpha}}\right)} \leq TE_{\max}$ …(6.2)

$=> \frac{1}{TE_{\max}}\sqrt{\hat{\bar{\alpha}}'\hat{\bar{\Psi}}^{-1}\hat{\bar{\Psi}}\hat{\bar{\Psi}}^{-1}\hat{\bar{\alpha}}} \leq \lambda$ …(6.3)

$=> \lambda \geq \frac{\sqrt{\hat{\bar{\alpha}}'\hat{\bar{\Psi}}^{-1}\hat{\bar{\alpha}}}}{TE_{\max}}$ …(6.4)

$\overline{w}^*$ can now be solved by substituting λ into equation 6.1.

As such, the portfolio with the maximum expected return (with the greatest permissible tracking error) is given by:

$$\overline{w}^* = \frac{TE_{\max}}{\sqrt{\hat{\overline{\alpha}}'\hat{\overline{\Psi}}^{-1}\hat{\overline{\alpha}}}}\hat{\overline{\Psi}}^{-1}\hat{\overline{\alpha}} \ldots(6.5)$$

Equation 6.5 has been derived by Lee and Lam (2001), and is presented in Scherer (2004) as well as Berkelaar *et al* (2006). Lee and Lam derived the formula using Lagrangian techniques, as opposed to the simpler method used here.

Interpreting equation 6.5

Equation 6.5 has intuitive appeal. Note that $\frac{TE_{\max}}{\sqrt{\hat{\overline{\alpha}}'\hat{\overline{\Psi}}^{-1}\hat{\overline{\alpha}}}}$ is a constant. If the alpha streams from each manager is independent of every other manager, then $\hat{\overline{\Psi}}$ is a square matrix with a diagonal consisting of elements $diag(\hat{\psi}_{11,t}, \hat{\psi}_{22,t}, ..., \hat{\psi}_{NN,t})$, and $\overline{C}$ reduces to the identity matrix. As such the inverse of $\hat{\overline{\Psi}}$ is just $diag(\frac{1}{\hat{\psi}_{11,t}}, ..., \frac{1}{\hat{\psi}_{NN,t}})$ and the vector $\overline{w}^* = \frac{TE_{\max}}{\sqrt{\hat{\overline{\alpha}}'\hat{\overline{\Psi}}^{-1}\hat{\overline{\alpha}}}} \cdot \begin{bmatrix} \left(\frac{\hat{\alpha}_{1,t}^{a_1}}{\hat{\psi}_{11,t}}\right) \\ \left(\frac{\hat{\alpha}_{2,t}^{a_1}}{\hat{\psi}_{22,t}}\right) \\ \ldots \\ \left(\frac{\hat{\alpha}_{N,t}^{a_1}}{\hat{\psi}_{NN,t}}\right) \end{bmatrix}$

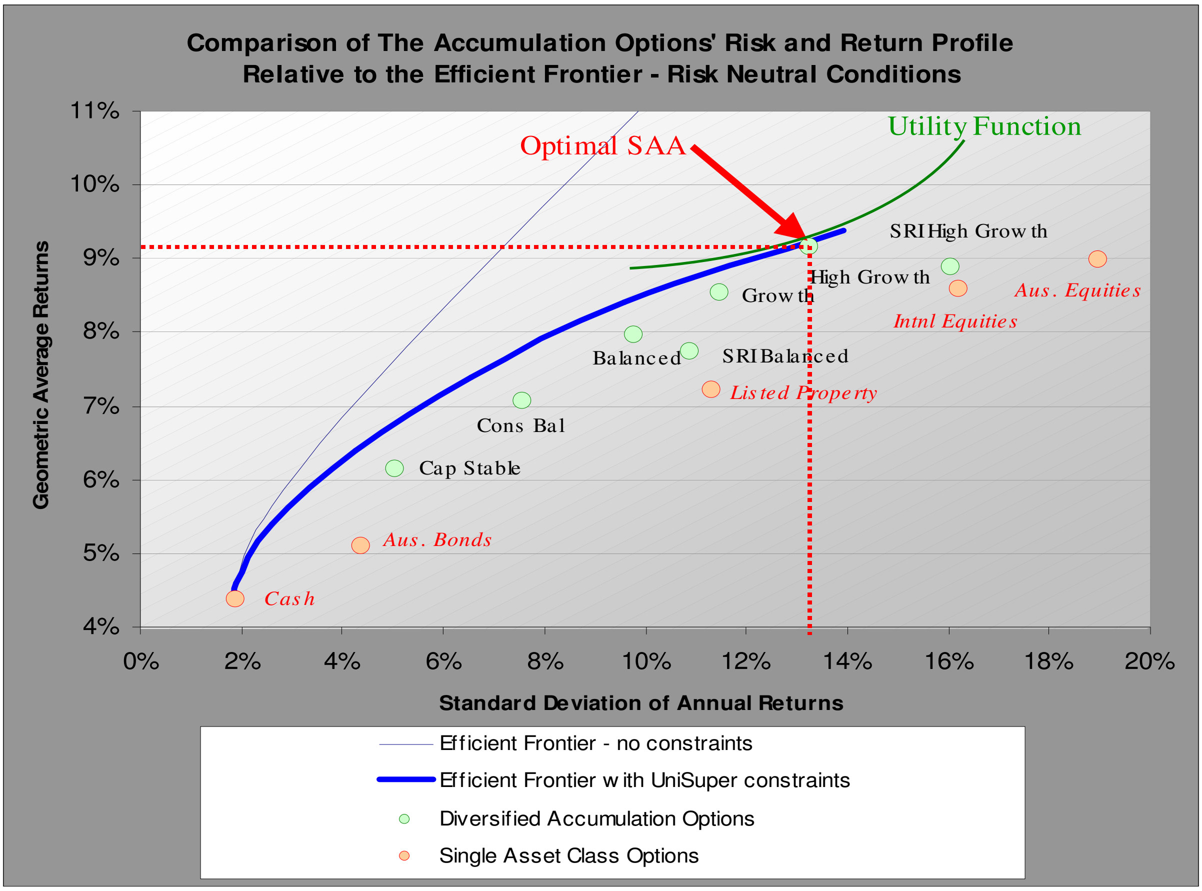
Hence the portfolio that maximises returns such that tracking error is limited to $TE_{\max}$ is proportional to the ratio of each manager's expected out-performance to the estimated variance of their excess returns. Equation 6.5 is an extension of most risk allocation models – except that correlations of excess returns between managers are not assumed to be zero.

A1.7. Incorporating the Fund's Strategic Asset Allocation (SAA)

Most risk budgeting frameworks are determined ignoring the Fund's liabilities (and hence SAA), typically by budgeting tracking error. UniSuper's risk budgeting framework differs to that used by most practitioners as TURBOs monitors the extent to which each Option deviates from its Strategic Asset Allocation (SAA).

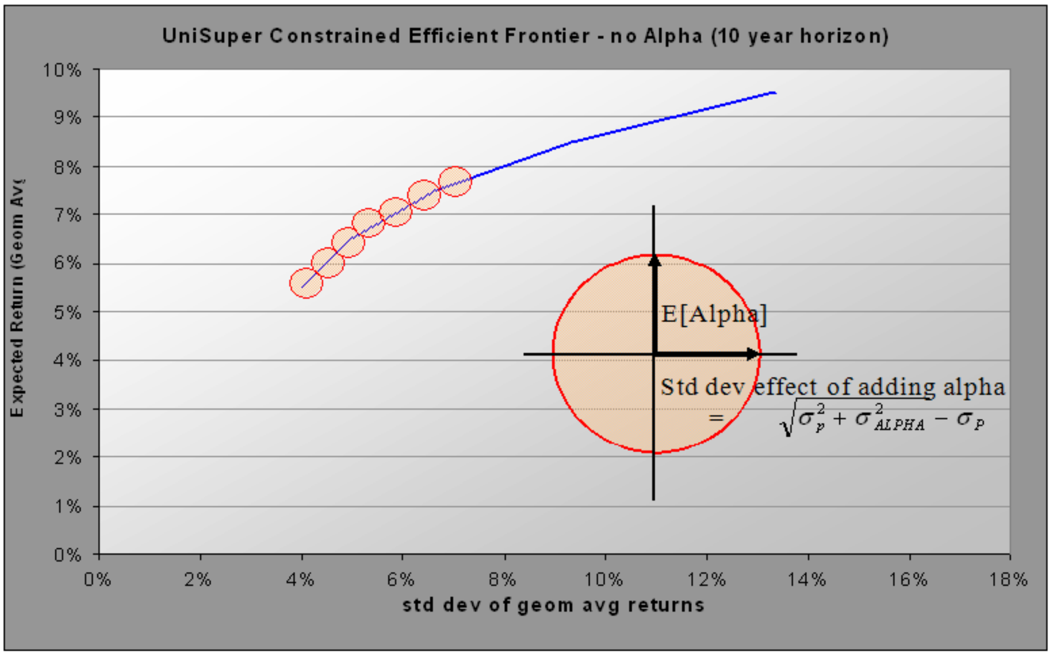
UniSuper's SAA for each option is set by applying a set of investment objectives along with investment constraints (such as removing the ability to short stocks, limiting exposure to alternative asset classes etc), and then assessing where on the constrained efficient frontier the Trustee would like the option to lie, by considering the balance between risk and reward.

Asset Liability Models generally consider only beta risk from each asset class. The thin dark blue curve in the chart below, demonstrates the Fund's unconstrained frontier, while the thicker light blue curve represents the Fund's constrained efficient frontier. Overlaying the Fund's investment objectives (possibly via the use of a utility function) allows the Fund to determine the optimal asset allocation for each option, as shown in the chart below for the Fund's High Growth Option [(4)].

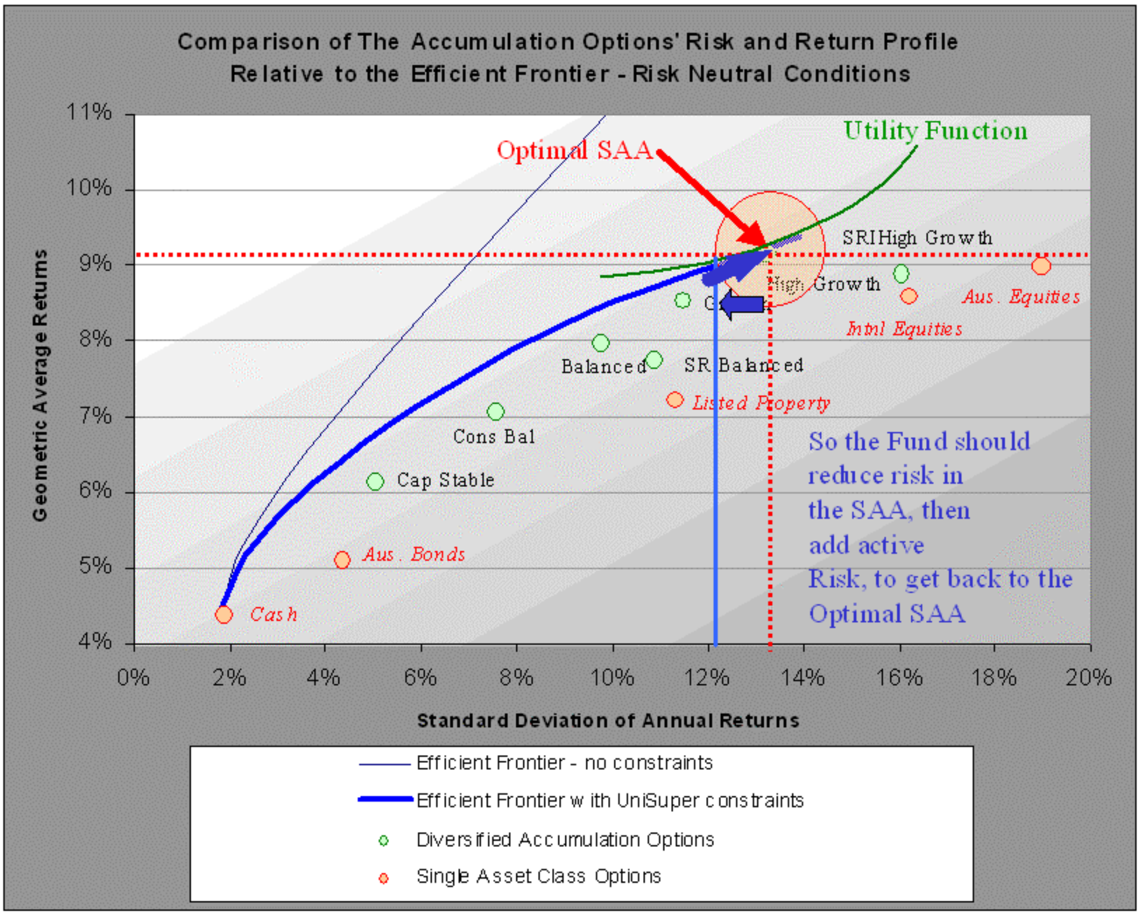


The Fund has the ability to invest passively and precisely match the beta exposures expected from each asset class. The extent to which the Fund employs active management represents a source of risk to the Fund.

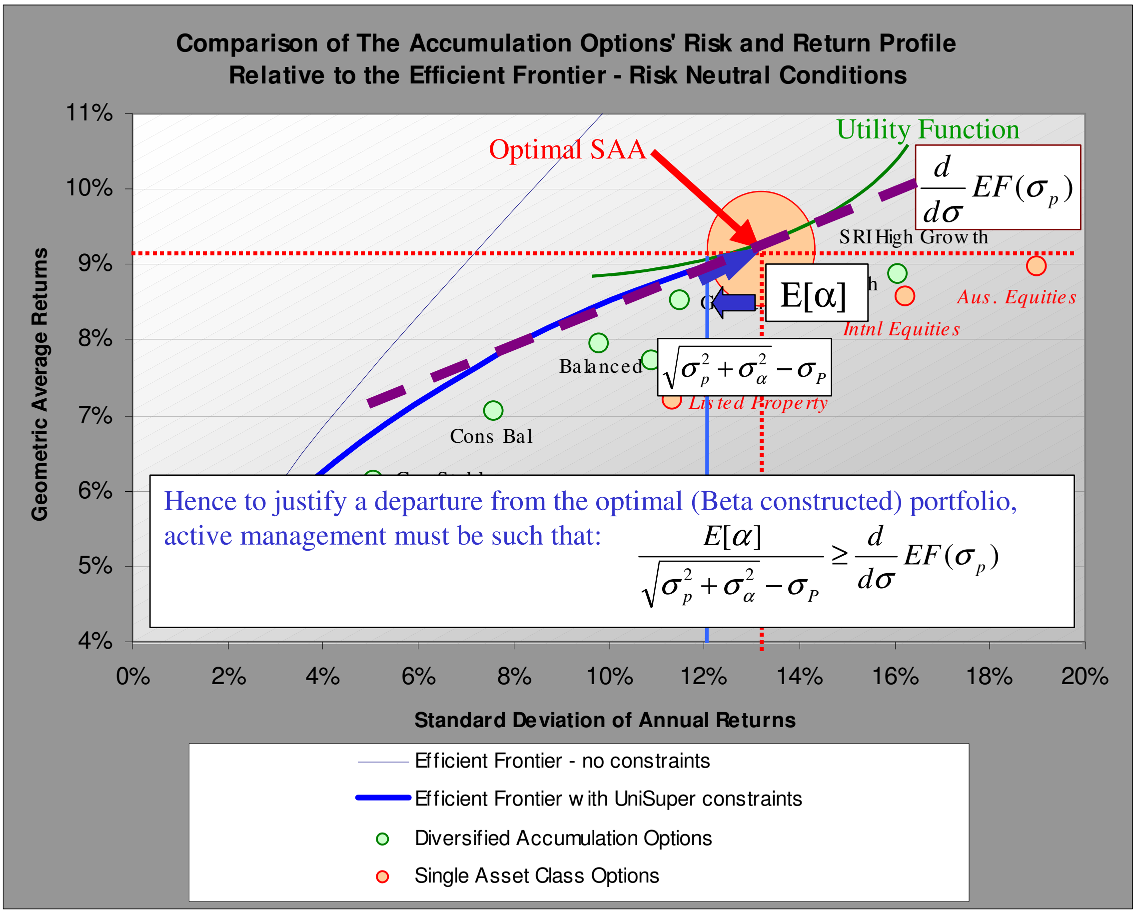
Consider the constrained efficient frontier, derived by investing passively (given by the thick blue curve above). Instead of investing passively, the Fund can invest actively, which adds the potential to improve returns (or dampen returns if managers are poorly selected). Additionally, such alpha sources can add risk to the portfolio, or possibly, reduce risk if the alpha source is uncorrelated to the beta returns. Graphically the addition of alpha could be thought of as increasing the "thickness" of the efficient frontier.



From what we can tell, Baars *et al* (2006) were the first to represent active returns as a circle in mean-variance space, we have extended their work to consider the impact of adding active risk to the efficient frontier. Effectively the optimal risk-return balance that was derived to meet the liabilities has been altered with the incorporation of active risk. To overcome this concern, the Fund could first remove beta risk (eg. by reducing the allocation to equities) and then add active risk, so as to re-establish the optimal SAA, as shown in the chart below.



If the Fund were to reduce its beta risk, and add active management to get back to the optimal SAA, and the active managers were able to generate an appropriate level of excess returns, then the Fund would be neutral to the decision to use active managers. This provides us with a key inequality that will be used in the risk budgeting formulation, namely that the option's ex-ante alpha needs to exceed a minimum level to justify a departure from beta allocations.



Using the approach outlined in section 6 we can extend the risk allocation problem to correspond with each option's strategic asset allocation. Once again we wish to maximise expected excess returns subject to the constraint that the ex-ante expected excess returns divided by the impact of the active management program on the Option's overall volatility is greater than the derivative of the efficient frontier at the Options' risk level.

Algebraically this problem can now be specified as:

Maximise: $\overline{w}'\hat{\overline{\alpha}}$, subject to the constraint that:

$$\frac{\overline{w}'\hat{\overline{\alpha}}}{\sqrt{\sigma_o^2+\overline{w}'\hat{\overline{\Psi}}\overline{w}}-\sigma_o} \geq \frac{\delta EF(\sigma_O)}{\delta\sigma}$$

$$\text{Or} \quad \overline{w}'\hat{\overline{\alpha}} \geq \left[\sqrt{\sigma_o^2+\overline{w}'\hat{\overline{\Psi}}\overline{w}}-\sigma_o\right]\left[\frac{\delta EF(\sigma_O)}{\delta\sigma}\right] \ldots (7.1)$$

Where σ_o^2 represents the variance of the option, based on the SAA long term assumptions and $\frac{\delta EF(\sigma_O)}{\delta\sigma} = \left.\frac{\delta EF}{\delta\sigma}\right|_{\sigma=\sigma_O}$ represents the derivative of the constrained efficient frontier, with respect to the volatility of the frontier, solved when $\sigma = \sigma_O$.

Recall that in section A1.1 we defined W^{o,a_i} as the time-independent Strategic Asset Allocation (SAA) weight for Option *o* within asset class a_i. Where $\overline{W_o}$ denotes the vector of Strategic weights to each factor exposure for Option *o*. In section A1.5 we defined $E\lfloor F_{k,t} \rfloor$ as the expected return for the Factor *k* at time *t,* while $\overline{F}$ denotes the vector of expected factor returns. Finally if $\bar{Z}$ denotes the covariance matrix of all benchmark factor returns ($BM^*_{m,t}$) utilised within the Fund's SAA, then $\sigma_O = \sqrt{\overline{W_o}'\bar{Z}\overline{W_o}}$

As $\frac{\delta EF(\sigma_O)}{\delta\sigma}$ and σ_O do not vary over time (unless the fund's SAA changes or the assumptions the Fund's long term assumptions are revised) both parameters are effectively constants, which vary for each Option.

Note that $\frac{\delta EF(\sigma_O)}{\delta\sigma}$ can be solved numerically using the formula:

$$\frac{\delta EF(\sigma_O)}{\delta\sigma} = Lim_{h\to 0}\left(\frac{EF(\sigma_O + h) - EF(\sigma_O)}{h}\right) \ldots(7.2)$$

While σ_O is derived from the Fund's SAA formulation. The values for $\frac{\delta EF(\sigma_O)}{\delta\sigma}$ and σ_O are shown in the table below, for each option based on UniSuper's latest SAA review:

UniSuper Accumulation Option (o)	Standard Deviation of Option Returns	Constrained Efficient Frontier		
		Maximum Return, under differing levels of risk		$\frac{\delta EF(\sigma_O)}{\delta\sigma}$
	$\sigma_O = \sqrt{\overline{W_o}'\bar{Z}\overline{W_o}}$	$EF(\sigma_O)$	$EF(\sigma_O + 0.1\%)$	
Cash	1.9%	4.381%	4.762%	**3.81**
Capital Stable	5.0%	6.753%	6.798%	**0.45**
Conservative Balanced	7.6%	7.750%	7.784%	**0.34**
Balanced	9.8%	8.430%	8.459%	**0.28**
Growth	11.5%	8.894%	8.920%	**0.26**
High Growth	13.3%	9.284%	9.299%	**0.14**

A1.8. Reverse Optimisation

From section 4.1 we know that the maximum of $\overline{w}'\overline{\mu} - \frac{\lambda}{2}\overline{w}'\overline{V}\overline{w}$ occurs, when the optimal portfolio (given by $\overline{w}^*$) equals $\frac{1}{\lambda}\overline{V}^{-1}\overline{\mu}$.

By substitution, the function $\overline{w}'\hat{\overline{\alpha}} - \frac{\lambda}{2}\overline{w}'\hat{\overline{\Psi}}\overline{w}$ is maximised if $\overline{w}^* = \frac{1}{\lambda}\hat{\overline{\Psi}}^{-1}\hat{\overline{\alpha}}$ …(7.2)

We now need to solve for λ such that the constraint in equation 7.1 is obtained.

Hence λ is such that: $\dfrac{\left(\frac{1}{\lambda}\hat{\Psi}^{-1}\hat{\alpha}\right)'\hat{\alpha}}{\sqrt{\sigma_o^2+\left(\frac{1}{\lambda}\hat{\Psi}^{-1}\hat{\alpha}\right)'\hat{\Psi}\left(\frac{1}{\lambda}\hat{\Psi}^{-1}\hat{\alpha}\right)}-\sigma_o} \geq \dfrac{\delta EF(\sigma_O)}{\delta\sigma}$

$$=> \frac{\frac{1}{\lambda}\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}}{\sqrt{\sigma_o^2+\left(\frac{1}{\lambda^2}\right)\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}}-\sigma_o} \geq \frac{\delta EF(\sigma_O)}{\delta\sigma}$$

$$=> \frac{1}{\lambda}\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}+\sigma_o\frac{\delta EF(\sigma_O)}{\delta\sigma} \geq \frac{\delta EF(\sigma_O)}{\delta\sigma}\sqrt{\sigma_o^2+\left(\frac{1}{\lambda^2}\right)\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}}$$

$$=> \frac{1}{\lambda^2}\left(\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}\right)^2+\frac{2}{\lambda}\sigma_o\frac{\delta EF(\sigma_O)}{\delta\sigma}\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}+\sigma_o^2\left(\frac{\delta EF(\sigma_O)}{\delta\sigma}\right)^2 \geq \left(\frac{\delta EF(\sigma_O)}{\delta\sigma}\right)^2\left[\sigma_o^2+\left(\frac{1}{\lambda^2}\right)\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}\right]$$

$$=> \frac{1}{\lambda^2}\left(\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}\right)^2+\frac{2}{\lambda}\sigma_o\frac{\delta EF(\sigma_O)}{\delta\sigma}\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha} \geq \left(\frac{\delta EF(\sigma_O)}{\delta\sigma}\right)^2\left[\left(\frac{1}{\lambda^2}\right)\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}\right]$$

$$=> \left(\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}\right)^2+2\lambda\sigma_o\frac{\delta EF(\sigma_O)}{\delta\sigma}\left(\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}\right) \geq \left(\frac{\delta EF(\sigma_O)}{\delta\sigma}\right)^2\left(\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}\right)$$

$$=> \left(\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}\right)+2\lambda\sigma_o\frac{\delta EF(\sigma_O)}{\delta\sigma} \geq \left(\frac{\delta EF(\sigma_O)}{\delta\sigma}\right)^2$$

$$=> \lambda \geq \frac{1}{2\sigma_o}\left(\frac{\delta EF(\sigma_O)}{\delta\sigma}-\left\{\frac{\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}}{\frac{\delta EF(\sigma_O)}{\delta\sigma}}\right\}\right) \quad \ldots(7.3)$$

The optimal portfolio $\overline{w}^*$ can now be solved by substituting λ from equation 7.3 into equation 7.2. As such, the portfolio with the maximum expected return (with the greatest permissible tracking error) is given by:

$$\boxed{\overline{w}^* = \frac{2\sigma_o\frac{\delta EF(\sigma_O)}{\delta\sigma}\hat{\Psi}^{-1}\hat{\alpha}}{\left(\frac{\delta EF(\sigma_O)}{\delta\sigma}\right)^2-\hat{\alpha}'\hat{\Psi}^{-1}\hat{\alpha}} \quad \ldots(7.4)}$$

Where $\hat{\alpha}$ is derived from equation 4.2.9 and $\hat{\Psi}$ is derived from equation 5.6.

Appendix 2 - Glossary of Symbols

General notation

Symbol	Denotes
o	An investment option or liability type (eg. Cash Option, Capital Stable Option, Defined Benefit Division etc).
a_i	The asset class of manager *i* (eg. Australian Equities, International Fixed Interest etc).
M	The number of asset classes that UniSuper utilises.
W_m^o	The Strategic Asset Allocation (SAA) weight for Option *o,* within asset class *m.* Note that *W* is time independent.
$\hat{\sigma}_{o,t}^2$	The estimated ex-post variance of Option *o* at time *t*.
$E[R_t^o]$	The expected (ex-ante) return from Option *o* at time *t*.

Notation relating to the estimation of each manager's factor exposures

Symbol	Denotes
$\hat{\beta}$	The CAPM beta factor describing the sensitivity between a security and the entire market. Also referred to as the systemic risk.
$\dot{w}_{i,t}^s$	Manager *i*'s holding in security *s* at time *t.*
S	The number of all available securities.
$\hat{\sigma}_{s,t}^{F_k}$	The estimate covariance between the security (*s*) and an appropriate market factor (F_k) at time *t.*
F_k	Various macro-economic factors or market indices, where sensitivity to changes in each factor is represented by a factor-specific beta coefficient ($\beta_{i,k,t}$) for a manager *i* at time *t*.
F^*	Residual factor *k* at time *t* (i.e. after removing the effects of higher order factors, to mitigate the impact of co-integration).
K	The number of all applicable factors (the risk free rate is an element of that set). Typical factors are returns on stock indices, interest rates, volatility etc.
k^*	The number of all significant regression factors obtained during a regression estimation.
$F_{k,t}$	The *observed* returns from factor *k* at time *t*.
$E[F_{k,t}]$	The expected return from factor *k* at time *t*.
$\beta_{i,k,t}$	The beta factor describing the sensitivity between manager *i*'s exposure to factor *k* at time *t*.

Factor notation

Symbol	Denotes
$BM^*{}_m$	The Fund's strategic benchmark, for the m^{th} asset class.
RP_k	The risk premium available from Factor *k* which cannot be attributed to one of the Fund's benchmarks.
$\hat{\gamma}_{m,k}$	The estimated sensitivity between factor *k* and the m^{th} asset class benchmark ($BM^*{}_m$).

Manager specific notation

Symbol	Denotes
$w_{i,t}^{a_i}$	The Fund's weight (as a proportion of total fund's under management) for manager *i* who operates in asset class a_i at time *t*. Note that *w* is option independent.
$w_{i,t}^{o,a_i}$	The Fund's weight within Option *o,* for manager *i* who operates in asset class a_i at time *t*.
$\overline{w}_t$	The vector of weights of all manager holdings in option *o* at time *t*. *ie* $\overline{w}_t' = \left[w_{1,t}^{o,a_1} \quad \dots \quad w_{N,t}^{o,a_N}\right]$
N	The total number of managers spanning all asset classes.
$\hat{\sigma}_{ij,t}$	The estimated covariance of net of tax and fees returns between manager *i* and manager *j* at time *t,* allowing for manager *i* and *j*'s auto-correlation factors.
$\hat{\rho}_{i,t}^{a_i}$	The estimated auto-correlation factor for manager *i* at time *t* .
$\overline{V}$	The variance-covariance matrix of total (net of tax and fees) returns for all *N* managers, at time *t*.
$\hat{\alpha}_{i,t}^{a_i}$	The estimated average idiosyncratic or non-diversifiable risk for manager *i* at time *t,* where manager *i* invests in asset class a_i.
$\hat{\beta}_{i,k,t}$	The estimated manager's average exposure to factor *k* at time *t*.
$\varepsilon_{i,t}$	The manager's residual error term or unexplained returns, with a zero mean.
r	The number of months' data utilised in the regression analysis in the assessment of each manager's factor exposures and idiosyncratic risk.
$\hat{\mu}_{i,t}^{a_i}$	The *ex-ante* expected total return (before tax, net of fees) from manager *i* at time *t* in asset class a_i.
$\hat{\overline{\mu}}$	The vector of *ex-ante* expected annual returns for each manager, at time *t*.
$\hat{\sigma}_{oi,t}$	The estimated covariance between manager *i* and Option *o* at time *t*.

Notation relating to the estimation of ex-ante Alpha

Symbol	Denotes
λ	An individual investor's risk aversion parameter.
$\dot{\lambda}^o$	The estimated risk aversion parameter for Option *o*.
N^*	The number of all available securities.
$\overline{w}_{eq}$	The equilibrium market capitalisation weights of all assets in the market (a 1 x *N** vector).
$\overline{\Sigma}$	The covariance matrix for all *N** assets.
τ	The scaling factor applied to $\overline{\Sigma}$ which measures <u>the uncertainty of the *priori*</u>
Π	the 1 x *N** vector of implied equilibrium returns (in excess of the risk free rate) for all securities.
δ	The world-wide risk aversion parameter.
$\overline{\mathrm{A}}$	The vector of observed ex-post alpha or excess returns for each manager.
$\mathrm{E}[\hat{\alpha}_{i,t}^{a_i}]$	The ex-ante expected alpha from manager *i*, at time *t*.
$\hat{\overline{\alpha}}$	The vector of each manager's expected ex-ante alphas.
$\hat{\overline{\Psi}}$	The covariance matrix of each manager's returns in excess of their factor exposures.
$\overline{Q}_A$	The investor's view of the expected alpha generated from each manager.
$\overline{\Omega}_A$	The investor's confidence in each manager's ability to generate alpha.

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Endnotes

(1) UniSuper is the industry super fund dedicated to all who work in Australia's higher education and research sector. With over 419,000 members and more than $23 billion in assets (at 30 June 2008), UniSuper is one of Australia's largest superannuation (i.e. pension) funds.

(2) We considered a range of smoothing periods as well as differing return lags, but concluded that a two-year smoothing of both manager/asset returns, as well as their associated risk factors, generated stable and interpretable results.

(3) Note the matrix notation utilised in this section, where vectors and matrices have been reduced to the constituent parts. As an example, $\overline{Y} = \begin{bmatrix} \overline{\Pi} \\ \overline{Q} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \Pi_1 \\ \dots \\ \Pi_{N^*} \end{bmatrix} \\ \begin{bmatrix} Q_1 \\ \dots \\ Q_N \end{bmatrix} \end{bmatrix}$.

(4) The standard deviations shown in the x-axis of the chart relate to annual standard deviations. When UniSuper considers the construction of each Option's strategic asset allocation, the Fund allows for the non-normality of returns over multiple time periods (a particular concern for asset classes with low annual volatility but with substantial positive serial correlation). As a result, the SAA for several Options don't lie on the constrained efficient frontier when considering annual standard deviation of returns, but are more efficient when one considers the option's time horizon and standard deviations over longer periods. However, it should be noted that the single asset classes (shown in red italics in the chart) offer less diversification than the Fund's pre-mixed Options, and hence are less efficient regardless of timeframe considered.